

***A new class of monthly rainfall-runoff models for ungauged basins –
Application to the Peruvian coastal basins***

Running title: A new class of monthly rainfall-runoff model for ungauged basins

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Abstract:

Rainfall–runoff modelling in data-scarce regions remains a major challenge in climatology due to limited hydrometric records and the heterogeneity of watershed responses. To address this problem, we introduce a new class of parsimonious nonlinear rainfall–runoff models that rely solely on two geometric descriptors of the basin: surface area and elongation ratio. The models are derived using the global modeling technique, insensitive to the initial conditions. Models are built and validated across 49 basins distributed over ~223,000 km² in the Peruvian Pacific slope and coast. Two generalized formulations were obtained: Model G_I , based on the four northernmost basins, and Model G_{II} , derived from a larger set. Both reproduce key hydrological features, including annual maxima, minima, and the geometry of phase-space orbits. Model G_{II} , although less parsimonious, consistently outperforms G_I in terms of error metrics, while retaining physical interpretability.

Simulations reveal that elongation exerts a stronger influence on runoff dynamics than area, particularly under extreme rainfall (e.g. El Niño events). Stability analysis further shows that basins with intermediate elongation (~0.45–0.60) are the most robust, whereas very rounded or highly elongated basins approach instability or divergence. Importantly, the proposed framework is transferable to ungauged basins, offering predictive capability with minimal parameterization.

These findings highlight the structural similarity of watershed responses across diverse climatic and geomorphic settings in coastal Peru, suggesting broader applicability to other semi-arid regions with episodic extreme events.

Keywords: basins runoff, global modelling technique, rainfall-runoff modelling, data-driven approach, ungauged basins

1. Introduction

Hydrological dynamics rely on numerous factors, both endogenous (such as topography, soil properties, surface vegetation, subsurface and underground hydrologic network, etc.) and exogenous (e.g., climatic forcing, groundwater pumping, irrigation, etc.). To generate accurate simulations of the hydrological dynamics requires detailed monitoring, which is not always possible or feasible.

Among the hydrological variables, runoff is of particular interest because it provides integrated information about the watershed dynamics as a whole. When both precipitation (typically the main input) and runoff are available, rainfall-runoff models can be very useful to get an overview of the other

contributions, particularly water use. Moreover, runoff is a key variable for water resources management and flood risk assessment.

Several types of monthly rainfall-runoff models have been developed for this purpose. On the one hand, mechanistic and conceptual rainfall-runoff models such GR2M, GR5M (Mouelhi et al., 2006), AWMW (Boughton, 2004), ABCD (Thomas, 1981), XM (Xiong and Guo, 1999), DWBM (Zhang et al., 2008) or the VUB (Vandewiele and Ni-Lar-Win, 2009) aim to represent the main or dominant processes at the basin scale. These approaches assume that the model structure is known and only require parameter calibration, which can be achieved from field observations. On the other hand, black box approaches (including Artificial Neural Networks, Support Vector Machine, and others) make few assumptions about system behavior. While highly versatile, these approaches often yield models that are difficult to interpret, as their ad hoc formulations are no more transparent than the original system (Riad et al., 2004; Raghavendra and Deka, 2014; Stefanoiu et al., 2024, among others). Meanwhile, alternative approaches have been developed under the assumption that, although the model structure is initially unknown, it can be meaningfully reconstructed. This can be achieved through libraries of structures associated with optimization techniques. For instance, libraries that incorporate a wide range of potential hydrological processes (Craig et al., 2020; Spieler et al., 2020) have been employed to rebuild model structures.

However, in many cases, data scarcity poses a critical challenge, and sometimes even runoff records are unavailable. This raises important questions regarding the generality and transferability of hydrological models when only limited information about watershed properties exists. To address this issue, alternative methods have been developed for ungauged basins, which represent a long-standing (Pickup, 1977) and challenging problem (Sivapalan, 2003; Wagener, 2004; Westerberg, 2016). For example, Castiglioni et al. (2009; 2010) used the multiple regression technique to relate rainfall with runoff in order to estimate the runoff in an ungauged basin, taking in account the drainage area, the percentage of permeable area, a synthetic description of the topography, the main channel length, the time of response, and the mean annual precipitation and temperature.

In this work, we focus on runoff and its relationship with the rainfall in 49 basins located along the Peruvian coastal slope and lowlands. Rainfall dynamics in South American coastal basins are governed by large-scale processes involving non-linear atmosphere-ocean interactions in the Pacific and at the global scale. These dynamics can produce both extreme rainfall excesses and deficits, with profound socio-economic impacts, particularly on agriculture and water security (Rodríguez-Morata et al., 2018; Moquet et al., 2018; Rau et al., 2017). Moreover, their high degree of unpredictability complicates modelling efforts.

Rau et al. (2017, 2019) provided a comprehensive analysis of climate and hydrology in 49 Peruvian coastal basins. Using statistical and wavelet analyses, they classified the region into nine climatic zones and compiled an extensive hydrometeorological dataset covering 1970–2019, which serves as the foundation for our work.

Here, we apply the global modelling technique for equation extraction. This mathematical method is capable of detecting nonlinear relationships without requiring strong a priori assumptions about model structure, parameterization, or underlying processes (Mangiarotti et al. 2025). Typically, it does not account for spatial variability in parameters. In our case, the method is applied to runoff dynamics at the basin scale. First, we derive nonautonomous Ordinary Differential Equations (ODEs) for each of the 49 watersheds. Next, we attempt a geographical generalization by incorporating basin area and shape, testing the approach on a subset of catchments before extending it to the full set. Our approach enables the estimation of runoff in ungauged basins using only precipitation, basin area, and basin elongation as input variables, whether derived from observed data or hypothetical scenarios. It should be noted that model parameterization requires only two parameters that can be directly derived from a Digital Terrain Model (DTM) of the basin, and the models are also computationally efficient. Furthermore, they enable the interpretation of hydrological processes a posteriori and the analysis of how these variables influence runoff across the study area.

2. Study Sites and Data

2.1. Region of Study and Watersheds

This study considers 49 distinct watersheds located along the Pacific slope of Peru (Figure 1). Each watershed extends from the Andean foothills to the Pacific Ocean, forming a continuous sequence from North to South. For reference, we label them B01, B02, ..., B49. These basins span a pronounced latitudinal climatic gradient: mean annual precipitation reaches 594.8 mm in the north (B01, Piura) but only 123.9 mm in the south (B49). Rainfall is concentrated in the mountains, and many rivers often dry before reaching the coast, even during winter.

In this region, two problems are particularly striking: droughts in the northern highlands during strong La Niña years, and floods in the middle and southern lowlands during strong El Niño years. El Niño and La Niña refer to two very specific situations of the coupling between ocean and atmosphere in the Pacific Ocean. El Niño situation occurs when surface temperatures in the eastern Pacific Ocean are warmer than average, shifting rainfall eastward toward South America, including Peru. Conversely, La Niña events push rainfall westward toward Asia, often resulting in drought across parts of Peru (Lavado-Casimiro et al. 2013). El Niño is a complex and irregularly periodic phenomenon whose pseudo-periodicity is variable

and highly unpredictable in terms of hydrological impact on the South American continent (Bourrel et al., 2015). During strong events, it couples with the annual continental cycle, producing heavy summer rains (Sanabria et al., 2018). However, the magnitude of El Niño does not always correspond directly to rainfall intensity at all latitudes (Lagos et al., 2008; Bourrel et al., 2015), reflecting a nonlinear atmosphere–ocean coupling.

Basin morphology also strongly influences the drainage system, and consequently, the runoff response (Sassolas-Serrayet et al., 2018; Rau et al., 2019). The relationship between rains and runoff can be described using the elongation ratio p_e (Sassolas-Serrayet et al., 2018). Based on this metric, we classified the 49 basins as follows: 24 are very elongated ($p_e < 0.5$), 23 are elongated (0.5–0.7), one is less elongated (0.7–0.8), one is oval (0.8–0.9); none approach the circular category (0.9–1.0). The most elongated basin is Olmos (B3), while the least elongated is Chilca (B27), and the only oval basin is Grande (B35). The drainage networks are generally symmetric, with some exceptions: the main stream lies near the northern boundary in B01, B34, and B35, and near the southern boundary in B45 and B47.

Basin size is another key factor, although the runoff–area relationship in dry lands is often complex (Tooth 2000; Reid et al. 2011). In our dataset, basin areas span nearly six orders of magnitude, ranging by a factor of ~28: from 620.3 km² in the Topara basin (B31) to 17,152.8 km² for Camana basin (B43), with a median of 3,062.6 km², corresponding to Chancay Huaral (B23) (Table 1). In this study, for clarity, we classified the basins into five size groups: very large (>15,000 km², two basins), large (10,000–15,000 km², two basins also), medium (5,000–10,000 km², three basins), small (2,500–5,000 km², 16 basins), and very small (<2,500 km², 23 basins). Larger and medium-sized basins are concentrated in the South, with the exception of Piura (B1) in the North and Santa (B13) in the central region. The smallest basins tend to be shorter, reflecting rivers that originate closer to the coast (e.g., Omas). Overall, basin area is roughly proportional to river length, except B35, B42, and B44, which are anomalously wide compared to their length.

2.2. Rainfall Dataset

The rainfall dataset consists of one representative time series per basin, coming from Rau et al. (2019), which yields a total of 49 time series with monthly data spanning 1970–2019, here extended to 1970–2023. These time series were constructed by Rau et al. (2019) to represent basin-wide precipitation, based on in-situ measurements from multiple stations within each basin and subsequently preprocessed to obtain a single representative record (Fig. S1–S3). For simplicity, we denote precipitation as ϖ , and its square root as P .

Minimum precipitation is close to zero in all basins (0.1–0.3 mm), reflecting dry-season conditions (Table S1). In contrast, median precipitation varies markedly among basins: Santa (B13), Chancay

Lambayeque (B05), and Jequetepeque (B08) are the wettest, with median values exceeding 45 mm a year. Nine basins show medians between 20 and 45 mm, the southernmost of which is Huaura (B22), while the remaining basins record medians below 20 mm. During extreme El Niño events, the basins with the highest maxima are not necessarily those with the largest medians. The clearest example is Piura (B1) (>500 mm), which has a relatively low median (14.4 mm) but records maxima above 500 mm, reflecting its strong sensitivity to El Niño. Eight other basins, all in the north (B02-B06, B08, B09 and B13), show maxima between 350 and 500 mm. In general, rainfall magnitude decreases from north to south, though exceptions exist—for instance, Ocona (B42) is relatively wet despite its latitude. Rainfall also tends to decrease from the mountains toward the coast, but since our analysis uses basin-averaged values, this gradient is not explicitly considered.

2.3. Runoff Dataset

One specific difficulty when comparing the runoff of different basins comes from the fact that, even when the basins are equipped, the gauges are not necessarily available at the river mouth. The runoff dataset used in this study, also derived from Rau et al. (2019) enables to overcome or at least to reduce this difficulty. It consists of 49 monthly time series of runoff covering 1970-2019 here extended to 1970-2023 (Fig. S4-S6) that are a byproduct of the conceptual GR2M model (Mouelhi et al. 2006), that is, a two-parameter monthly water balance model. Its parameter calibration was performed considering seven basins on the periods 1970-2005 (B01, B16), 1970-2006 (B32, B43) and 1970-2008 (B09, B36, B45). Its parameterization was then generalized considering three characteristic parameters: the basin area, the main channel length and the basin perimeter. The runoff was then estimated at the river mouth of each of the 49 basins by running the model using these local parameters. For this reason, this product represents a spatially and temporally extended version of the original records, recalibrated and validated against in-situ hydrometric stations to estimate runoff at each river mouth, regardless of the gauging station location. This approach facilitates direct comparison among basins. Anthropogenic influences were minimized by using only semi-natural gauged basins as signals for calibration and validation (see Rau et al, 2018). For simplicity, runoff is denoted as ρ , and its square root as R .

Although monthly precipitation is generally scarce in the study area, minimum values are usually above zero (Table S1). In contrast, median runoff at the river mouths is often zero or near zero. Only a few basins show higher median runoff: Santa (B13, 13.8 mm), Ocona (B42, 8.4 mm), and Chancay Lambayeque (B05, 6.2 mm). These basins, along with Pativilca (B20) and Cañete (B30), also display relatively high third quartiles (15-45 mm). The remaining basins exhibit much lower values, with medians below 3.5 mm and third quartiles between 5-15 mm (19 basins) or 0-5 mm (25 basins).

The influence of El Niño on basin hydrology is clearly expressed in the runoff maxima. The Piura basin (B01) records the highest maximum (469.8 mm), consistent with its strong sensitivity to ENSO-driven rainfall. The next largest maxima are observed in Santa (B13) and Chancay Lambayeque (B05), both around 275 mm. Several other basins—B02, B04, B06, B08, B09, B10, as well as Pativilca (B20), Cañete (B30), and Ocoña (B42)—exhibit maxima between 175 and 250 mm, the latter three being notable given their more southerly location. In general, runoff tends to decrease from north to south, though the pattern is not systematic. Unlike precipitation alone, runoff variability reflects the combined influence of rainfall, basin geometry, evapotranspiration, and other hydrological processes, resulting in irregular spatial distributions.

2.4. Pre-processing

While higher-resolution data could provide additional detail, such data are not available for this large temporal and spatial extent, and their inclusion would likely increase model complexity at the expense of interpretability. Therefore, the original monthly time series were upsampled to 12 values per month. Without this refinement, the coarse timestep may hinder the estimation of the derivatives and compromise the reconstruction. Upsampling also enhanced the ability to capture nonlinearities of the system. We employed Stineman interpolation (Stineman, 1980), which minimizes the number of inflection points and, thus, avoids the artifacts commonly introduced by spline interpolation (Zhang and Martin, 1997). The procedure occasionally generated small negative values, which were subsequently set to zero.

No additional smoothing was required beyond the implicit filtering performed by the global modelling code during the estimation of the derivatives. This algorithm uses a default window of nine timesteps (equivalent to approximately three weeks), whose effect on the data was marginal. It was necessary to apply a square root transformation to the data, as searches using the original variables produced no results—likely due to the large disparity between dry and wet years. We actually tested three approaches—log transformation, no transformation, and square root transformation—but only the square root transformation produced numerically stable models. All models were therefore constructed using the transformed variables ($R = \rho^{0.5}$ and $P = \varpi^{0.5}$). Model errors, however, were evaluated before and after back-transforming to the original runoff variable (l). The errors of the squared runoff are more representative of the low-flow season, while the original runoff better represents high-flow events. For visualization, the root-transformed series (R and P) were occasionally retained.

Finally, the time series were divided into two periods: from 1 January 1970 to 10 April 2003 for model calibration, and from 12 April 2003 to 1 December 2019 for independent validation.

3. Theoretical Background

This study focuses on two main variables: runoff (ρ), which is the primary variable of interest, and rainfall (ϖ), considered as the main forcing. Although causally related, these variables exhibit distinct dynamics. Rainfall is governed by deterministic physical processes but is characterized by high dimensionality and stochastic-like variability (von Storch, 2004). Runoff, by contrast, reflects the integrated effect of rainfall over the basin and thus displays smoother, lower-dimensional dynamics (Mangiarotti et al., 2019). Because the influence of the El Niño phenomenon is already embedded in precipitation records, including sea surface temperature in the models offered no added predictive value (on the contrary, it only increased complexity).

3.1. The Global Modelling Technique

The global modelling technique takes its background from the theory of nonlinear dynamical systems, or theory of chaos (Bergé et al. 1984). This theory enables to understand how a system can be both deterministic and unpredictable in the long term, due to high sensitivity to initial conditions. The global modelling takes advantage of this theory by developing a technique independent to the initial conditions. It is designed to extract dynamical equations directly from time series and also applies to chaotic dynamics (Aguirre and Billings, 1995; Gouesbet and Letellier, 1994). Initially tested to theoretical cases, it was successfully applied in the early 1990s to single experimental time series in electrochemistry and mixing reactors (Letellier et al., 1995; Letellier et al., 1997). More recently, it has been extended to multiple observational time series from real-world systems (Mangiarotti and Huc 2019, Mangiarotti 2015). The approach has also been shown to handle multiple time series taken in association (Mangiarotti et al., 2016) and to model the dynamics of non-autonomous systems (Sáez et al., 2021; 2024).

In this work, two formulations are considered. The first is a local (ad hoc) formulation:

$$dR_r/dt = F(R_r, U_r(t)) + \varepsilon \quad (1)$$

where R_r denotes the modelled variable (here, the square root of runoff) and $U_r(t)$ represents the forcing (here, the square root of precipitation, P), explicitly dependent on time t , for the basin r , centred at coordinates (λ, φ) , F is a function and ε a dynamical noise assumed to be Gaussian and centred. The second is a generalized formulation

$$dR_r/dt = F(R_r, U_r(t), p_a, p_e) + \varepsilon \quad (2)$$

where p_a and p_e are basin-specific geographical parameters corresponding to area and elongation ratio, respectively. This new formulation aims to generalize the usual global modelling technique. In most of the previous studies, this technique was applied locally, or, at best, assuming that the spatial variability could be resumed as a single dynamic, either by aggregating the observations as a homogeneous signal, or by considering the time series in association, that is, as an ensemble of dynamics possibly independent (in phase, in initial conditions) but all identical (same equations and parameterizations). This problem was investigated both theoretically by considering ensembles of Rössler systems (Mangiarotti et al. 2016) or practically for the cycles of cereal crops observed by satellite in Morocco (Mangiarotti and Le Jean 2023). The formulation presented here in Eq. (2) is still independent from the initial conditions and the forcing, but more general by reconstructing not only the original set of equations but also parameter dependence. This is what makes this approach a new class of global models because it aims at reproducing not only the global dynamics as a particular case, but as general case taking also in account the variations of its parameters. Of course, only the role of the parameters provided as an input can be expected to be involved in the reconstructed set of equations. The parameters p_a and p_e are incorporated into the generalized formulation (Eq. 2) to model multiple basins simultaneously, whereas the local formulation (Eq. 1) targets individual basins and does not include p_a or p_e explicitly.

In practice, the GPoM R package was used to obtain a polynomial approximation of the function F , both in terms of structure and parameterization, following the procedure presented in Mangiarotti and Huc (2019), with adaptations to account for multiple basins when constructing generalized models. The same modelling period was applied to all the time series, spanning from 1 January 1970 to 10 April 2003. All analyses were performed using the GPoM package (Mangiarotti et al., 2023), which is freely available from the Comprehensive R Archive Network (CRAN): <https://CRAN.R-project.org/package=GPoM>.

3.2. Adaptations to the Study of Generalized Models

Some methodological modifications were introduced to derive generalized models, i.e., single formulation models applicable to multiple basins. These models capture broader dynamics, as they are derived from data collected from multiple basins. To achieve this, time series for all basins involved in the modelling were considered simultaneously in the modelling process. To do so, the time series of each variable were concatenated sequentially to create a single extended precipitation time series. Basically, concatenation was performed by assembling the time series of the modelling basins one after the other, as illustrated in Figure 2 (here considering four modelling basins). In the present case, we need four time series: the first one for $R(t)$, the second one for $P(t)$, the third one for $p_a(t)$ and the fourth one for $p_e(t)$. This way, each variable includes the concatenated information of the four basins used for the modelling (B01, B02, B03 and B04, respectively in purple, pink, cyan and blue colours). For each basin, the area

and the elongation parameters are constant in time, but they vary spatially from one basin to another. A gap was inserted between each consecutive basin series in order to avoid spurious values in the derivatives. The combined time series were used as input for the GPoM function. Model validation and error estimation were performed independently for each basin, using the validation periods, before and after reversing the square-root transformation.

First, a generalized model ($\mathbf{G}_I$) was developed using the first four basins (Piura to Motupe) which differ considerably in size and shape but are relatively dry on average (see Table S1). This model was found applicable to most of the 49 basins, with varying but generally acceptable performance.

To improve representativeness, a second generalized model ($\mathbf{G}_{II}$) was then constructed using basins distributed across all latitudes of the study region. Specifically, the modelling process was applied to every third basin in the sequence—i.e., the 1st, 4th, 7th, ... up to the 49th—resulting in a set of 17 basins. This choice was applied systematically, no specific basin selection was operated a posteriori in the modelling process. The remaining 32 basins were reserved for validation.

Note that the modelling and validation periods, and the modelling and validation basins are all synthesized in Table 2. There was no ad hoc decision to include or exclude any catchments: these choices were applied systematically and were not changed in the modelling-validation process.

3.3. Adaptations to Handle the Number of Possible Models

In the original GPoM algorithm, the model search for a two-dimensional system (one equation for the runoff and one for the precipitation) with maximum polynomial degree of three, yields 2^{20} possible model structures. Indeed, for two variables (here R and P), equations can have 10 terms each, which makes 2^{10} each, and then 2^{20} for the two equations. The GPoM algorithm includes two stages: a first stage makes a drastic structure pre-selection, but without fostering smaller or larger equation sizes. On the contrary, it keeps a full range of possibilities for each equation, that is, from all the terms (here 10 terms) down to no term at all, which gives here eleven possible structure for each equation, and therefore $11 \times 11 = 121$ possibilities for the two equations taken together. The robustness of these preselected models are then tested by numerical integrability in the second stage.

In the present study, however, we deal with a non-autonomous equation for the root runoff (Eq. 1), that is, an equation explicitly forced by an external variable $P(t)$ (the root precipitation). Thus, it is neither necessary nor feasible to model an equation for precipitation with a high precision: its dynamic is high-dimensional, while GPoM is designed for low-dimensional systems. To account for this, we adapted the code to restrict the search to the runoff equation only. This modification substantially reduced the number of possible model structures to be tested ($N = 2^{10}$ for a single equation of two variables, here P and R , with maximum polynomial degree of 3).

For the generalized models, the number of possible model structures increases substantially because four variables are considered instead of two, which also raises the maximum polynomial degree from 3 to 5. This expansion results in $N^* = 2^{126}$ possible model structures, compared to N for the individual models. Indeed, although the watershed geometric parameters (p_a and p_e) are ultimately used as coefficients, they must be introduced during model search as stepped time series ($p_a(t)$ and $p_e(t)$) together with $R(t)$ and $P(t)$. In the individual models, the maximum polynomial degree was limited to 3. In the generalized case, however, each term may also be multiplied by p_a , p_a^2 , p_e , p_e^2 and $p_e p_a$. Since the maximum polynomial degree of R and P combined is 3 and the maximum polynomial degree of p_a and p_e combined is 2, the total maximum polynomial degree reaches 5. For example, the term RP^2 expands into six possible variants: RP^2 , $p_a RP^2$, $p_a^2 RP^2$, $p_e RP^2$, $p_e^2 RP^2$, and $p_a p_e RP^2$.

Finally, we applied a template derived from the individual models, discarding all the terms that did not appear in any of them.

3.4. Validation

At this point, each model search retrieves multiple individual models based on the same template and it is necessary to select the best for each basin. The models were validated by numerical integration over the validation window, using the observed root precipitation as forcing. The same validation period was applied to all time series, from April 12, 2003, to December 1, 2019. First, we calculated the correlation coefficient between data and predictions, as well as the RMSE for each model. Second, we selected the model with better performance for each basin (the individual model with better performance for basin 1, the individual model with better performance for basin 2, etc.; the generalized model with better performance for basins 1 to 4 etc.)

3.5 Application to other basins and comparison

These global error measurements, however, have some limitations when it comes to comparing the performances of either multiple models on the same basin or a single model on multiple basins. Because the original variables were squared for modelling, the errors tend to increase as the runoff values become larger. Thus, a reasonable RMSE for an extreme event (e.g., 10 mm) can exceed the annual maximum runoff in a year of scarce precipitation, even within the same basin. Similarly, accurate reproduction of extreme events may yield an overly optimistic correlation coefficient. To better assess model performance across different hydrological conditions, these errors were computed for each hydrological cycle (September–August). The statistics of the annual RMSEs were, then, derived for each basin and model. As noted above, RMSE generally increases with runoff maxima. To normalize this effect and enable inter-basin comparison, we defined a relative maximum RMSE, obtained by dividing the maximum RMSE by

the maximum observed runoff. This metric provides a measure of the relative performance of each model during extreme events. Finally, we also analyzed the RMSE calculated directly from the root square runoff in order to get an error that is less sensitive to the high events and more appropriate to assess low-flow and intermediate-flow conditions.

4. Results

Because rainfall occurs mainly in the upper watershed, runoff data were preprocessed to reflect conditions at the basin outlet rather than at the mid-basin gauge stations. This adjustment was performed using a regional hydrological model based on GR2M (Mouelhi et al., 2006), which aggregated precipitation across the entire basin (Rau et al., 2019). Therefore, our global models capture rainfall forcing and its downstream response—after losses by infiltration, evaporation, and other processes. In fact, in many basins and years, rivers may dry out completely before reaching the ocean. Anthropogenic influences are, in principle, excluded thanks to the preprocessing procedure of Rau et al. (2018).

4.1. Individual Models

An individual model was obtained for each of the 49 basins, resulting in 19 different model structures (Table 3), labelled from **a** to **s**. None of the individual models includes a constant term or a linear precipitation (P) term; these are, therefore, omitted in Table 3. Structures **a**, **d**, **f**, **g**, **h**, **i**, **j**, **k**, **l**, **n**, **o**, and **r** (39 models) are all mutually compatible in their structure: terms may be present or absent, but shared terms always have the same sign. In contrast, structures **e**, **m** and **p**, (6 models) are incompatible with the previous group, as they contain positive RP and R^2P terms. Structures **q** and **s**, corresponding to two southernmost basins (models for B47 and B49), are unique, and compatible with each other. Finally, structures **b** and **c** (respectively for B03 and B04 models) are unique, but **c** is compatible with both **b** and the **e-m-p** group, whereas **b** is not (see Table 3 caption for details).

For the individual models, global RMSE values on the validation period (2003-2023) range from 0.08 mm (B27) to 10.34 mm (B01), with a median of 3.02 mm (Table S2). These values are generally lower than those of the generalized models, since the calibration focused specifically on each basin. Correlation coefficients vary between 0.82 (B46) and 0.98 (B05), with a median of 0.96.

Analysis of the RMSE across hydrological cycles yields further insight into model accuracy (Table S3). Piura basin (B01) exhibits the highest 90% percentile RMSE (19.63 mm) and the second highest maximum RMSE (25.88 mm). However, because this basin experiences the largest runoff maxima due to the strong El Niño influence, its relative maximum RMSE is only 5.5%, indicating highly accurate

predictions (Figure 3). In contrast, Chilca (B27) shows the lowest maximum RMSE (0.28 mm). Despite a small maximum runoff of 1.78 mm, the relative maximum RMSE is 15.6%. Nevertheless, the model accurately predicts the extreme dryness and minimal runoff of this basin (Figure 3). These examples highlight the importance of considering both absolute and relative RMSE when evaluating model performance.

The lowest-performing individual models correspond to Motupe (B04) and Ocona (B42). Motupe (B04) exhibits the highest maximum RMSE (35.94mm) with moderate maxima, resulting in a relative maximum RMSE of 15.3%. Despite this, the model successfully distinguishes between low, medium, and extreme events (Figure 3). Ocona (B42), a large Southern basin, ranks third in 90% percentile RMSE (11.46 mm) and seventh in maximum RMSE (12.05 mm), with a maximum runoff of 84 mm. Its relative maximum RMSE is 14.2%, indicating moderate performance, with some overestimation of a few annual maxima (Figure 3). The largest relative error is observed in B39, at 22.2% (Figure 3).

The RMSE computed from the root precipitation P ranges from $0.08 \text{ mm}^{0.5}$ for B27 to $0.92 \text{ mm}^{0.5}$ for B01. Basin B35 RMSE represents only the 3.3% of the maximum recorded root runoff (during the validation period) and basin B42, the 10.02%. Overall, the results indicate good performance in absolute and relative terms.

4.2. Generalized Models G_I

Model G_I was derived using only the first four basins – the most northern ones (B01 to B04) – which differ considerably in size and shape. Despite these differences, in latitude, shape and size, the generalized model obtained from the four northern basins performs well across most of the 49 basins, indicating that their general hydrodynamic structure is compatible. The generalized model

$$\dot{R} = (\theta_1 p_e - \theta_2 p_e^2) R P^2 - \theta_3 R^2 - (\theta_4 - \theta_5 p_e) R^2 P \quad (3)$$

is parsimonious, comprising five terms that reduce to three when the elongation ratio p_e is fixed. Notably, the area parameter p_a was not retained (although it was included in the search), indicating that basin shape (elongation) is more influential than size within this group of four basins. Structurally, generalized models resemble the individual structure **a**, with terms in RP^2 , R^2 , and R^2P , but it also incorporates information about basin shape, allowing adaptability to basins whose individual structures differ (e.g., Olmos and Motupe, corresponding to structures **b** and **c**).

The coefficient of RP^2 is always positive for elongation ratios between 0.3 and 0.674, typical of most basins except the more rounded ones: Chilca (B27) and Grande (B35). The R^2P coefficient is negative for basins with elongation ratios lower than 0.546 (that is, for basins B01-B03, B05, B06, B08, B09, B13,

B14, B18, B20-B22, B24-B26, B28, B30-B34, B36, B39, B41, B43, B45-B47 and B49), that is, almost two thirds of the basins. Finally, the R^2 term is consistently negative and independent of basin geometry.

Overall, G_I can be applied to almost all the basins, with varying performance. Remarkably, a model based on four northern basins approximates the dynamics of basins across the entire study area, despite the latitudinal and climatic gradient. Performance is more irregular than for individual models, reflecting the limited calibration base. Notably, the model ignores the basin area, as it was rejected by the algorithm despite large size differences among the four basins.

Figure 4 plots each basin maximum RMSE against its relative maximum RMSE, with the line $y = -0.002x + 0.281$ (where x is the maximum RMSE and y is the relative maximum RMSE), separating poor (above the line) from acceptable performance (below). Some large southern basins (B35, B37, B38, B42 and B44) exceed 30% relative maximum RMSE, making the model G_I unsuitable. Certain small middle basins (B12, B15, B16, B23) also perform poorly (maximum RMSE >25 mm; relative RMSE >25%). In some very dry basins, the maxima are underestimated but the overall performance is moderate because their maximum RMSE is still low. For the rest of the basins, combinations of absolute and relative maximum errors allow the application of model G_I . For example, most basins with considerable runoff – such as those located in the north and under the direct influence of El Niño (B01-B03, B05, B06, B08, B09) – have maximum RMSE between 15-35mm but their relative maximum error is always under 10%. Very dry basins (such as B39, B40, B48 and B49) have considerable relative maximum RMSE but still very low maximum. Finally, the basins with better performance (maximum RMSE <12mm and relative maximum RMSE <12%) are B03, B08, B21, B28 and B45; these basins are limited neither to the modelling basins nor to the North, instead they cover the whole latitudinal range.

These results are confirmed by the RMSE computed from the root runoff, which ranges from 0.20 mm^{0.5} for B27 to 2.29 mm^{0.5} for B16. For a few basins (B12, B15, B16, B23, B35 and B42, already discarded in the previous paragraph) the RMSE is equal to or larger than 1.5 mm^{0.5} and all of them correspond to basins with low flow where the model G_I fails to reproduce the maxima.

4.3. Generalized Model G_{II}

The generalized model G_{II} (Eq. 4)

$$\begin{aligned} \dot{R} = & \beta_0 R P^2 + R^2 (\beta_1 - \beta_2 p_e + \beta_3 p_e^2) - R^2 P (\beta_4 - \beta_5 p_e + \beta_6 p_e^2 - \beta_7 p_a + \beta_8 p_e p_a + \beta_9 p_a^2) \\ & + R^3 (\beta_{10} - \beta_{11} p_e + \beta_{12} p_e^2 - \beta_{13} p_a + \beta_{14} p_e p_a + \beta_{15} p_a^2) \end{aligned} \quad (4)$$

was derived from basins B01, B04, B07... to B49. Compared to G_I , it is less parsimonious, as it includes the area p_a , but it achieved lower overall errors (see Table S2-S3). The model contains sixteen terms,

which reduce to only four once p_a and p_e are specified for a given basin. The resulting simplified structure corresponds to structure **a**, with the addition of an R^3 term (see Table 3).

Model G_{II} can be applied to most basins, though with some exceptions. Predictions diverge for basins B03, B27, B35, and B49 (hence they are absent from Figure 4B). Moreover, for basins B42 and B44, the annual maxima are strongly underestimated—similar to the behaviour of model G_I —while for B41, the model fails to reproduce the annual seasonality correctly. Basins B17, B34, and B40 show only moderate performance: they present high relative maximum RMSEs ($>25\%$) but low absolute maximum RMSEs (<15 mm). For the remaining basins, however, model performance is satisfactory. The RMSE calculated from the root runoff ranges from $0.24 \text{ mm}^{0.5}$ for B48 to $2.09 \text{ mm}^{0.5}$ for B42. The latter, along with B41, has a root runoff RMSE larger than $1.5 \text{ mm}^{0.5}$ confirming the problems mentioned above for these basins. Overall, model G_{II} outperforms G_I in terms of absolute RMSEs, as illustrated in Figure 4, where the cloud of points in the right panel (G_{II}) shift clearly to the left compared to the left panel (G_I). Relative maximum errors are also lower for G_{II} , with fewer basins located in the upper part of the right-hand plot. Although a large portion of the diagram remains empty, the same reference line used for G_I can still be applied here, effectively distinguishing poor (orange and red) from good (green) performance. Only two basins, B38 and B39, fall above this line. In the green-performance area, the only basins with maximum RMSE exceeding 25 mm are B01 and B05, yet both retain relative maximum RMSEs below 10%. This outcome is expected, as these northern basins experienced strong maxima during the El Niño event included in the validation window.

4.4. Phase portraits

The phase portrait projections of precipitation versus runoff describe anticlockwise, lance-shaped orbits (Figure 5A-5B). Smaller orbits are more horizontal, while the larger ones more oblique. For medium to high events, three distinct slopes emerge as precipitation and runoff increase. At low precipitation (<25 -50 mm, corresponding to 5 - $7.5 \text{ mm}^{1/2}$ in the plots), runoff remains close to zero, producing a nearly flat slope. The second slope is clearly positive but moderate, while the third is steeper, marking the sharpest runoff response. After the annual maximum is reached, the orbits descend in a single stretch, though the slope sometimes changes gradually.

By construction, differential phase portraits (R , dR/dt) are clockwise. They reveal further diversity of shapes across basins. For example, basin B01 (Figure 6A, grey line) displays an approximately triangular shape with three different slopes. First, runoff increases gradually from near 0 towards its maximum (upper left side of the triangle). In the second slope, the runoff continues to raise until its peak, but the growth rate slows and may even turn slightly negative. This second slope may be gradual (as in B01), but in many basins it changes more abruptly. Finally, runoff decreases slowly, with the rate shifting from negative to

near zero, creating an almost horizontal segment along the triangle's base. Large events (larger orbits) typically produce more irregular trajectories than the frequent cycles and clearly exhibits quick runoff decreases when its values are high. By contrast, other basins such as B09 (Figure 6B, grey line) exhibit more symmetric, drop-shaped orbits with smoother transitions.

All three models (individual, $\mathbf{G}_I$, and $\mathbf{G}_{II}$) reproduce the overall size and shape of the orbits in the phase space with accuracy (Figure 5). The precipitation versus runoff plots (Figure 5A-5B) capture the annual maximum runoff, the descending slope, and the annual minimum reasonably well.

Differences arise, however, in the representation of the ascending slopes: the model orbits transition more smoothly than the original data, producing slightly wider orbits compared to the sharp lance-shaped curves in the data. In the models, the two ascending slopes are connected by a gradual transition instead of an abrupt inflection, and the annual maximum is reached and left in a smoother manner. This behaviour is expected, since the models are based on the derivative of the runoff and, therefore, cannot reproduce the abrupt changes seen in the data. A finer temporal resolution of the original rainfall and runoff records (currently one month) could potentially improve the ability of the models to capture these faster slope variations and likely improve predictive performance, particularly for estimating extreme events. However, it would also increase model complexity, making the results more difficult to interpret. For this reason, we preferred to reserve the analysis of higher-resolution data for future work.

5. Discussion

Most of the data-driven approaches will produce black-boxes which interpretation can be very limited. The global models here obtained have an algebraic structure which can be analyzed to interpret the hydrological processes, a posteriori. The interpretation can even go further, here, because the equations also include parameters (here the basins area and elongation) that are characteristic of the hydrological basins.

In general, the models contain three types of terms: those depending only on precipitation P (P^2 and P^3), those depending only on runoff R (R , R^2 , and R^3), and mixed terms involving both P and R (RP , R^2P , and RP^2). Terms depending solely on P capture the effect of precipitation as an external forcing, contributing to watershed recharge, and, therefore, they are expected to be positive. In contrast, terms depending only on R reflect the emptying of the watershed (an autonomous process independent of precipitation) and they are expected to be negative. Mixed terms, involving both P and R , reflect the interaction between rainfall and the current hydrological state of the watershed, showing that watershed

dynamics are not solely determined by precipitation, but also by the current state of the basin. These mixed terms can be either positive or negative.

For the individual models, the terms in P (P^2 and P^3) are consistently positive, as expected (Table 3). Terms in R and R^2 are always negative, while the term in R^3 is negative in all cases except for structures **q** (basin B47) and **s** (basin B49). For basin B47, the sum of the R terms remains, nevertheless, negative, whereas basin B49 shows an overall positive global contribution from R terms, which can be regarded as an anomaly. This specific characteristic may be the result of very local effects associated with dams or irrigation systems, whose effects might be difficult to remove because of the narrowness and relatively small size of these two basins: additional information would be required. Groundwater flows resulting from the complex geology of this area may have an influence on the contribution of runoff as a source rather than a sink. The RP^2 term, linear in R and nonlinear in P , might represent the dominant effect of precipitation and it is accordingly, positive in almost all basins, except for structure **b** (basin B03). Finally, the RP and R^2P terms can be negative (30 basins), positive (8 basins), or absent (11 basins). When both RP and R^2P are present, their signs are always consistent.

The generalized model $\mathbf{G}_I$ lacks terms depending solely on the external forcing, i.e., precipitation. It includes a nonlinear term in R (R^2), independent of basin geometry and consistently negative, as expected. The RP^2 term is nonlinear in the elongation ratio p_e and positive, except for the most rounded basins (B27 and B35). In these cases, $\mathbf{G}_I$ predictions converge to zero, correctly describing the minimal runoff of B27 but underestimating the small peaks of B35. For all other basins, RP^2 remains positive. The R^2P term, linear in p_e , captures the interaction between the internal dynamics and the external input: it contributes to the recharge in less elongated basins ($p_e > 0.546$) and to the discharge in more elongated basins ($p_e < 0.546$). The contribution of the term R^2P is smaller in magnitude compared to the other terms and sometimes it is close to zero (e.g., basins B18 and B25). As a result, the combined contribution of R^2P and RP^2 is always positive (Table S4).

Model $\mathbf{G}_{II}$ includes two nonlinear runoff terms (R^2 and R^3). The R^2 term is always negative, while the R^3 term is negative for nearly all basins, with the exception of B05, B12, B16, B21, B37 and B41–B44. Despite these exceptions, the combined contribution of R^2 and R^3 remains negative for all basins (except B41), as expected, since these terms represent the emptying of the watershed. The term RP^2 is consistently positive, reflecting its contribution to the watershed recharge. By contrast, the R^2P term is negative for 26 basins (B01, B02, B07–B11, B14–B20, B22, B25, B28–B32, B34, B39, B40, B46–B48) and positive for the remainder. Nevertheless, the joint effect of R^2P and RP^2 is positive for all basins except B41.

Thus, model $\mathbf{G}_{II}$ is coherent for 48 basins, with two terms representing the emptying of the watershed (R^2 and R^3) and two terms capturing the interaction between the external input and the internal dynamical

state of the water flow (R^2P and RP^2) (Table 4). Therefore, in both generalized structures (G_I and G_{II}), the contributions to runoff variations can be separated in two main components: a positive contribution (R^2P and RP^2) and a negative one (R^2 and R^3).

Figure 7 displays the distribution of basins according to their area and elongation ratio, classified into several groups. The largest group (A, cyan) corresponds to basins with intermediate elongation ratios ($\sim 0.45-0.60$) and areas up to $12,000 \text{ km}^2$ approximately. In this group, the positive RP^2 term is dominant, exceeding the also positive contribution of R^2P , while the negative effect of R^2 is stronger than that of R^3 , although all contributions remain significant (Table S5). Surrounding this cluster lies a ring of basins (group B, purple) characterized by negative R^2P values. At more extreme values of area and elongation ratio, the R^3 term becomes positive (group C, magenta), but the combined effect of R^2 and R^3 remains negative. In these basins (with the exception of B37), the dominant contributions come from R^2P and RP^2 , though R^2 and R^3 also play non-negligible roles. Finally, for basins with the most extreme values of area or elongation ratio, the models tend to become unstable (group D, orange, including B41) or even divergent (group E, red).

For model G_{II} , the R^2 term depends nonlinearly on the elongation ratio (p_e) and is independent of the basin area, whereas the R^3 term depends nonlinearly on both p_e and p_a . For these terms, either:

$$\beta_2 p_e > \beta_1 \text{ and } \beta_2 p_e > \beta_3 p_e^2 \text{ and } \beta_{11} p_e > \beta_{10}$$

or:

$$\beta_{12} p_e^2 > \beta_{13} p_a > \beta_{14} p_e p_a > \beta_{15} p_a^2.$$

The contribution of $R^3(-\beta_{13} p_a + \beta_{15} p_a^2)$ is always negative and the contribution of $R^3(\beta_{14} p_e p_a)$ is always positive, with their sum often close to zero (Table S6). Consequently, the dependency on basin area is generally secondary. Indeed, in many basins with elongation ratios between $\sim 0.5-0.6$ (B01, B04, B07, B10-12, B14, B15, B17-19, B22, B23, B25, B26, B29, B31, B38-40, B46-48), the contribution of $R^2(\beta_1 - \beta_2 p_e + \beta_3 p_e^2) + R^3(\beta_{10} - \beta_{11} p_e + \beta_{12} p_e^2)$ is clearly dominant ($\geq 90\%$) compared to $R^3(-\beta_{13} p_a + \beta_{14} p_e p_a + \beta_{15} p_a^2)$. This indicates that the basin area plays a significant role in the R^3 term only when the basin shape is considerably more rounded or elongated than average. The RP^2 term is independent of basin geometry, while the R^2P term depends nonlinearly on both p_e and p_a . For RP^2 , $\beta_5 p_e > \beta_4$ or $\beta_6 p_e^2 > \beta_7 p_a > \beta_8 p_e p_a > \beta_9 p_a^2$. The contribution of $-R^2P(-\beta_7 p_a + \beta_9 p_a^2)$ is always positive and the contribution of $-R^2P(\beta_8 p_e p_a)$ is always negative (opposite to what happened with R^3). These contributions are clearly larger than $-R^2P(\beta_4 - \beta_5 p_e + \beta_6 p_e^2)$, but they cancel each other to a great extent and their sum is of the same order as $-R^2P(\beta_4 - \beta_5 p_e + \beta_6 p_e^2)$. Sometimes they cancel each other almost completely, as in basin B01.

Simulations of model G_{II} were performed under multiple areas and elongation ratios to better understand the role of parameters p_a and p_e (see Appendix A for details). Figure 8 illustrate how the basin response differs between dry/moderate years and the extremely rainy year, when a strong El Niño event got coupled with the Peruvian hydrological cycle. As expected, smaller elongation ratios ($p_e = 0.4$ km/km) produce larger annual peaks (purple line) because the water gathered at different points of the basin reaches the river sooner. Conversely, for the more rounded basin (magenta line) the peak is smaller. During the strong El Niño event, increasing the elongation ratio from 0.51 to 0.65 km/km markedly reduced the annual maximum runoff (from ~90 mm to ~55 mm), whereas decreasing it to 0.4 km/km raised the maximum but not more than in dry and moderate years. In both cases, the hydrograph tails lengthen: in more rounded basins, the peak is smaller and water release is, thus, slower; in more elongated basins, water reaches the river rapidly, but the discharge cannot occur instantaneously, extending the tail.

The effect of basin area is less pronounced than that of the shape. Expanding the area from 4,334 to 7,000 km² produces little change in runoff, which is common in dry basins where rainfall from distant areas seldom contributes significantly to river flow. Even reducing the area to 600 km² decreases the peak only slightly. However, under the strong El Niño event, this pattern is reversed: larger basins amplify the peak, whereas reducing the area has little effect.

The analysis of the stability condition (see Appendix A.2 for details) showed that for average area and varying elongation ratios (Figure 9a), differences among basins become more pronounced as precipitation is increased. At $P^2 = 600$ mm, the runoff ranged from ~50 to ~500 mm, with most basins (80%) exceeding ~400 mm. For average shape and varying areas (Figure 9b), the response was more complex. For precipitation up to 400 mm, all basins followed similar trends. Beyond this point, differences among basins widened, with runoff ranging from 450 to 875 mm at 600 mm of precipitation. This indicates that basin area significantly influences runoff, but primarily under high precipitation conditions, which is consistent with the results of Section 5.1.

Figure 10 shows the precipitation required to sustain specific runoff levels (50, 100, 200, ... 500 mm). The white areas surrounding the colored zones indicate precipitation values above 900 mm, whilst the white tongues inside the colored zone indicate points where the discriminant is negative. In order to maintain (in a hypothetical static situation) a runoff of 50 mm, most basins require precipitations around 130 mm. However, basins B03, B35, B42, B44, and B49 require substantially higher precipitation, reflecting the known limitations of model G_{II} for these basins.

A tongue of points outside the stability range appears on the left. As the target runoff level increases, this tongue shrinks and subsequently reappears on the right, leaving a few basins (B42, B44) outside the stability range. The precipitation required to maintain higher runoff levels also increases, roughly 210 mm for 100 mm of runoff, 325 mm for 200 mm, and 600 mm for 500 mm. As the target runoff rises, more

basins—such as B27 and B41—enter the region where required precipitation escalates sharply, near the edge of the coloured zone. The slight leftward tilt of the coloured stability zone indicates that small basins are slightly more stable when they are rounded, whereas larger basins are more stable when they are elongated. Thus, the analysis of these patterns provided a basis for interpreting the underlying hydrological processes and rejecting unsuitable model structures.

6. Conclusions

This paper introduces a new class of rainfall-runoff models directly built from observational time series and basin characteristics. These models constitute a new class of rainfall-runoff models because (1) they are global models: their aim is not to fit the time series but to reproduce the original dynamic as a whole, directly from particular time series, and independently to the initial conditions; and (2) because they enable to extract interpretable equations — including interpretable parameters characteristic of the studied (hydrological) systems. The obtained models depend only on two geometric parameters of the basin – area and elongation ratio – and can therefore be applied to ungauged watersheds. The models are parsimonious, involving two categories of nonlinear terms: (1) interaction terms that couple rainfall input with the current hydrological state of the watershed, and (2) autonomous terms that represent the internal dynamics of runoff.

These models are based either on single basins, or on multiple basins considered simultaneously, in association (specifically model G_I is based on four basins and model G_{II} on seventeen). Both demonstrate wide applicability across basins covering an area of ~223,000 km² in the Peruvian Pacific slope and coast, provided that basin areas and elongation ratios remain within moderate ranges (mainly small to medium, moderately elongated basins). This indicates that, beyond differences in rainfall regimes, watershed responses in this region follow a remarkably consistent dynamical pattern, based on catchment morphology.

These results highlight the potential of this modelling framework as a practical tool for hydrological prediction in data-scarce regions. Future work should test the transferability of these models to other geographic contexts, particularly to arid basins subject to infrequent but extreme rainfall events, in order to assess their robustness under broader hydrological conditions.

Conflict of interest statement

We have no conflict of interest to disclose.

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Data availability statement

The data that support the findings of this study can be made available on reasonable request.

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Appendix A: Simulations

In order to analyse how watershed responses vary with area and elongation ratio under different rainfall conditions, we performed simulations with the generalized model $\mathbf{G}_{II}$ using multiple forcings and different parameterizations of p_e and p_a . The structure of this model (Eq. 4) and its predictive performances will be considered in Section 5. Before that, our objective is to investigate the model response under different basin parameterizations and forcing conditions (Section A.1) and to characterize its stability (Section A.2).

A.1 Real forcings

We selected basin B22 as reference for this analysis because it is intermediate in terms of area ($p_a = 4,334 \text{ km}^2$), shape ($p_e = 0.51 \text{ km/km}$), precipitation (Table S1), and latitude (Figure 1). Six hydrological years were considered: three with average precipitation (2013–2014, 2014–2015, and 2018–2019), the first and third showing monomodal rainfall distributions and the second a bimodal one; two with low annual precipitation (2015–2016 and 2017–2018); and one with very high precipitation (2016–2017).

We first investigated the parameter range of validity. The model for basin B22 was found stable for areas between 600 and 7000 km^2 and elongation ratios between 0.4 and 0.65 km/km . Simulations were, then, performed to examine the influence of p_e and p_a on runoff under different rainfall conditions, first fixing p_a to 4,334 km^2 and varying p_e across its extremes, and then fixing p_e at 0.51 km/km while varying p_a .

A.2 Stability condition

We define the stability condition as the precipitation level required to maintain the discharge at a given level. This condition can be directly deduced from equation (4) by imposing $\dot{R} = 0$, which corresponds to equilibrium in the runoff dynamics. In this case, the relation reduces to the precipitation P needed to sustain equilibrium:

$$P = f(R, p_e, p_a), \quad (5)$$

and, conversely, to the precipitation needed:

$$R = f(P, p_e, p_a). \quad (6)$$

The stability condition was evaluated for basin areas ranging from 0 to 18,000 km^2 and elongation ratios between 0 and 1. Although the calculations involve R and P , the final results are presented in terms of runoff ρ (R^2) and precipitation ϖ (P^2).

830 First, only one parameter was varied while keeping, the other unchanged ($p_a = 4,334 \text{ km}^2$ or $p_e =$
831 0.51 km/km). Second, simulations were also performed varying both parameters under different
832 precipitation levels.

833 TABLES:

834 Table 1. Basic geometrical traits of the forty-nine basins considered in the analysis.

Number	Abrv.	Name	Area p_a (km ²)	Length (km)	Elongation ratio p_e (m ² /m ²)
B01	PIU	Piura	11,019.4	236	0.50
B02	CSJ	Cascajal	3,993.3	156	0.46
B03	OLM	Olmos	1,082.3	125	0.30
B04	MOT	Motupe	3,694.3	114	0.60
B05	CHL	Chancay_Lambayeque	4,061	180	0.40
B06	ZAN	Zaña	1,762.8	108	0.44
B07	CHM	Chaman	1,356.4	75	0.55
B08	JEQ	Jequetepeque	3,968.8	149	0.48
B09	CHI	Chicama	4,529.5	153	0.50
B10	MCH	Moche	2,131.9	93	0.56
B11	VIR	Viru	1,926.3	85	0.58
B12	HMN	Huamansana	1,439.5	65	0.66
B13	SAN	Santa	11,661.5	295	0.41
B14	LCR	Lacramarca	842.5	63	0.52
B15	NEP	Nepena	1,889.2	81	0.61
B16	CAS	Casma	2,990.7	95	0.65
B17	CLB	Culebras	670.9	53	0.55
B18	HMY	Huarmey	2,245	98	0.55
B19	FOR	Fortaleza	2,353	95	0.58
B20	PAT	Pativilca	4,601.7	160	0.48
B21	SUP	Supe	1,021.2	88	0.41
B22	HUA	Huaura	4,333.9	145	0.51
B23	CHH	Chancay_Huaral	3,062.6	100	0.62
B24	CHIL	Chillon	2,222.3	123	0.43
B25	RIM	Rimac	3,503.9	123	0.54
B26	LUR	Lurin	1,642.5	98	0.47
B27	CHLC	Chilca	783.4	45	0.70
B28	MAL	Mala	2,332.1	118	0.46

B29	OMA	Omas	1,117	65	0.58
B30	CAN	Canete	6,049.4	179	0.49
B31	TOP	Topara	620.3	58	0.48
B32	SJN	San Juan	3,353.2	135	0.48
B33	PIS	Pisco	4,231.2	172	0.43
B34	ICA	Ica	7,340.8	210	0.46
B35	GRN	Grande	11,049.9	138	0.86
B36	ACA	Acari	4,316	175	0.42
B37	YAU	Yauca	4,322.6	119	0.62
B38	CHA	Chala	1,232.4	65	0.61
B39	CHP	Chaparra	1,282.3	82	0.49
B40	ATI	Atico	1,400.3	71	0.59
B41	CRV	Caraveli	1,956.7	131	0.38
B42	OCO	Ocona	15,998.1	230	0.62
B43	CAM	Camana	17,152.8	355	0.42
B44	QUI	Quilca	13,548.6	220	0.60
B45	TMB	Tambo	13,073.4	295	0.44
B46	MOQ	Moquegua	3,419.1	130	0.51
B47	LOC	Locumba	5,862.2	175	0.49
B48	SAM	Sama	4,641.6	138	0.56
B49	CPL	Caplina	919.7	105	0.33

835

836

837 *Table 2. Periods and basins used for calibration and validation.*

838

<i>Model structure</i>	<i>Data period</i>	<i>Calibration period</i>	<i>Validation period</i>	<i>Validation</i>	
				<i>Calibrated</i>	<i>Uncalibrated</i>
<i>Individual</i>	1970-2023	1970-2003	2003-2023	<i>All the basins:</i> 1-49	0 -
G_I	”	”	”	4 basins: 1-4	45 basins: 5-49
G_{II}	”	”	”	17 basins: 1,4,7,10,13,16,19, 22,25,28,31,34, 37,40,43,46,49	32 basins: 2,3,5,6,8,9,11,12, 14,15,17,18,20,21, 23,24,26,27,29,30, 32,33,35,36,38,39, 41,42,44,45,47,48

839

840

841 Table 3. Structures of the individual (*a* to *s*) and generalized (**G_I** and **G_{II}**) models. Ten basins from Piura
842 (B01) to Moquegua (B46) share the same structure **a**, which is a parsimonious formulation with only
843 three terms. Structure **d**, also found in ten basins spanning a wide latitudinal range, is similar to **a** but it
844 replaces the negative R^2 term with a negative RP term. Structure **o**, shared by Ocona (B42) and
845 Camana (B43), the two largest southern basins, includes a P^2 term but otherwise resembles structure **a**.
846 Structure **i** occurs in six basins from Casma (B16) to Caraveli (B41) and it is a simplified version of **a**,
847 lacking the term R^2P . Structure **j** (three basins) and **k** (two basins) are very similar, differing by only one
848 term. All of these structures (**a**, **d**, **o**, **i**, **j**, **k**) are mutually compatible: terms may be present or absent,
849 but shared terms always have the same sign. Structures **f**, **g**, **h**, **l**, **n**, and **r** occur in single model each
850 but remain compatible with this group. In contrast, structures **e** (three models), **m** (two models) and **p**
851 (one model) are incompatible with the previous group, as they contain positive RP and R^2P terms.
852 Structures **q** and **s**, corresponding to two southernmost basins (B47 and B49), are unique, but
853 compatible with each other. Similarly, B03 and B04 exhibit structures **b** and **c**, respectively; here **c** is
854 compatible with both **b** and the **e-m-p** group, whereas **b** is not.

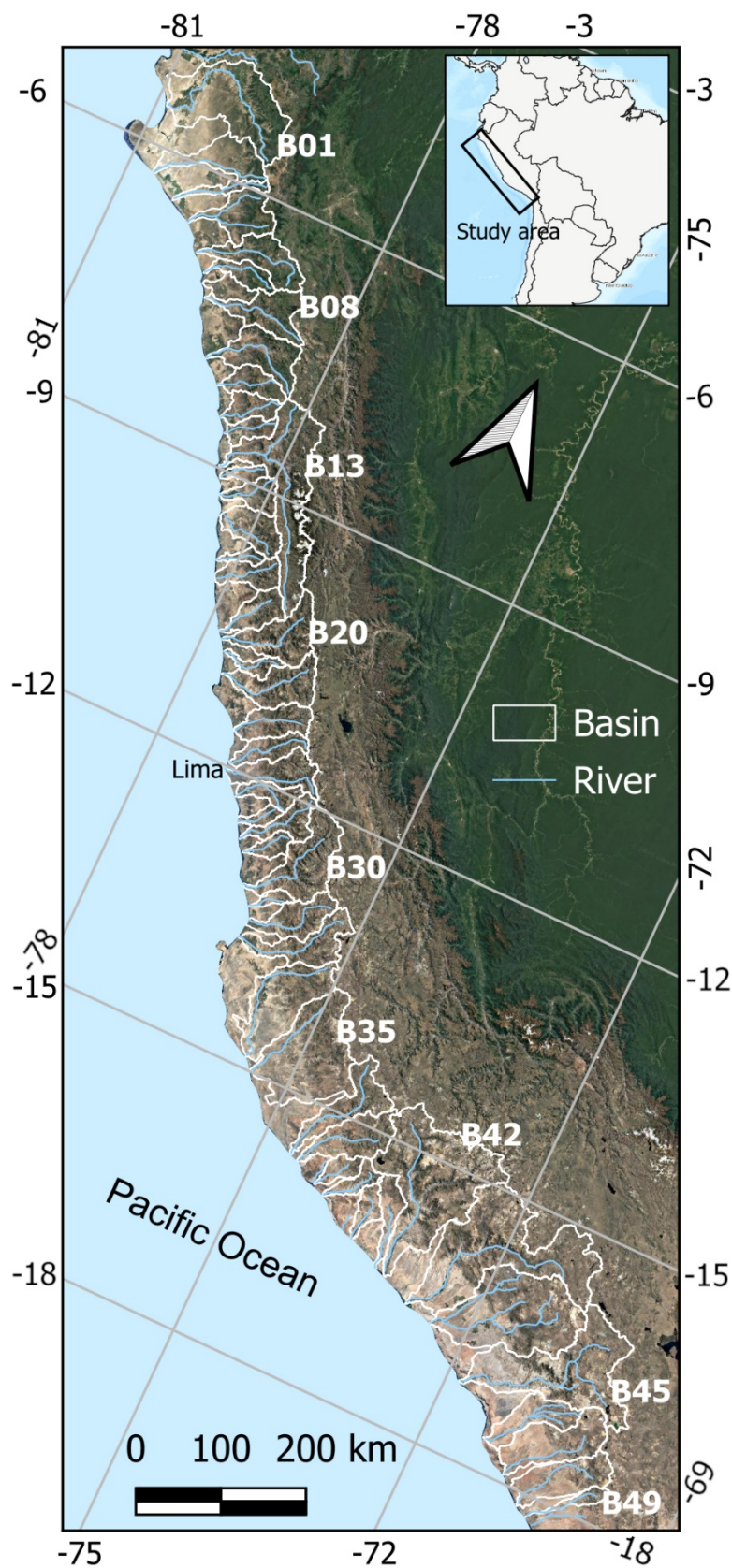
Individual models									
Structure	Monomials								Basins
	P^2	P^3	R	RP	RP^2	R^2	R^2P	R^3	
<i>a</i>					+	−	−		1,2,8,9,23,28,30,35,45,46
<i>b</i>		+	−	+	−	−	+	−	3
<i>c</i>				+		−	+	−	4
<i>d</i>				−	+		−		5,6,12,14,20,24,26,31,33,37
<i>e</i>					+	−	+	−	7,10,17
<i>f</i>				−	+	−		−	11
<i>g</i>					+	−		−	13
<i>h</i>				−	+	−	−		15
<i>i</i>					+	−			16,19,22,25,38,41
<i>j</i>		+		−	+			−	18,32,34
<i>k</i>		+		−	+		−		21,29
<i>l</i>		+	−						27
<i>m</i>				+		−	+		36,39
<i>n</i>		+		−	+				40
<i>o</i>	+				+	−			42,43
<i>p</i>				+	+	−			44
<i>q</i>					+	−	−	+	47
<i>r</i>		+		−	+				48
<i>s</i>				−	+		−	+	49
Generalized models									
Structure	Monomials								Basins
	P^2	P^3	R	RP	RP^2	R^2	R^2P	R^3	
G_I					+	−	+		1,4,7,8,9,10,11,12,14,16,17,18
					+	−	−		2,3,5,6,13,21,24,33,36,41,43,45,49
					−	−	+		27, 35
G_{II}					+	−	−	+	5,12,16,21,37,41:44
					+	−	−	−	4,6,13,23,24,26,33,36,38,45
					+	−	+	−	1,2,7:11, 14, 15, 17:20, 22, 25
					D/C	D/C	D/C	D/C	3,27,35,49

855

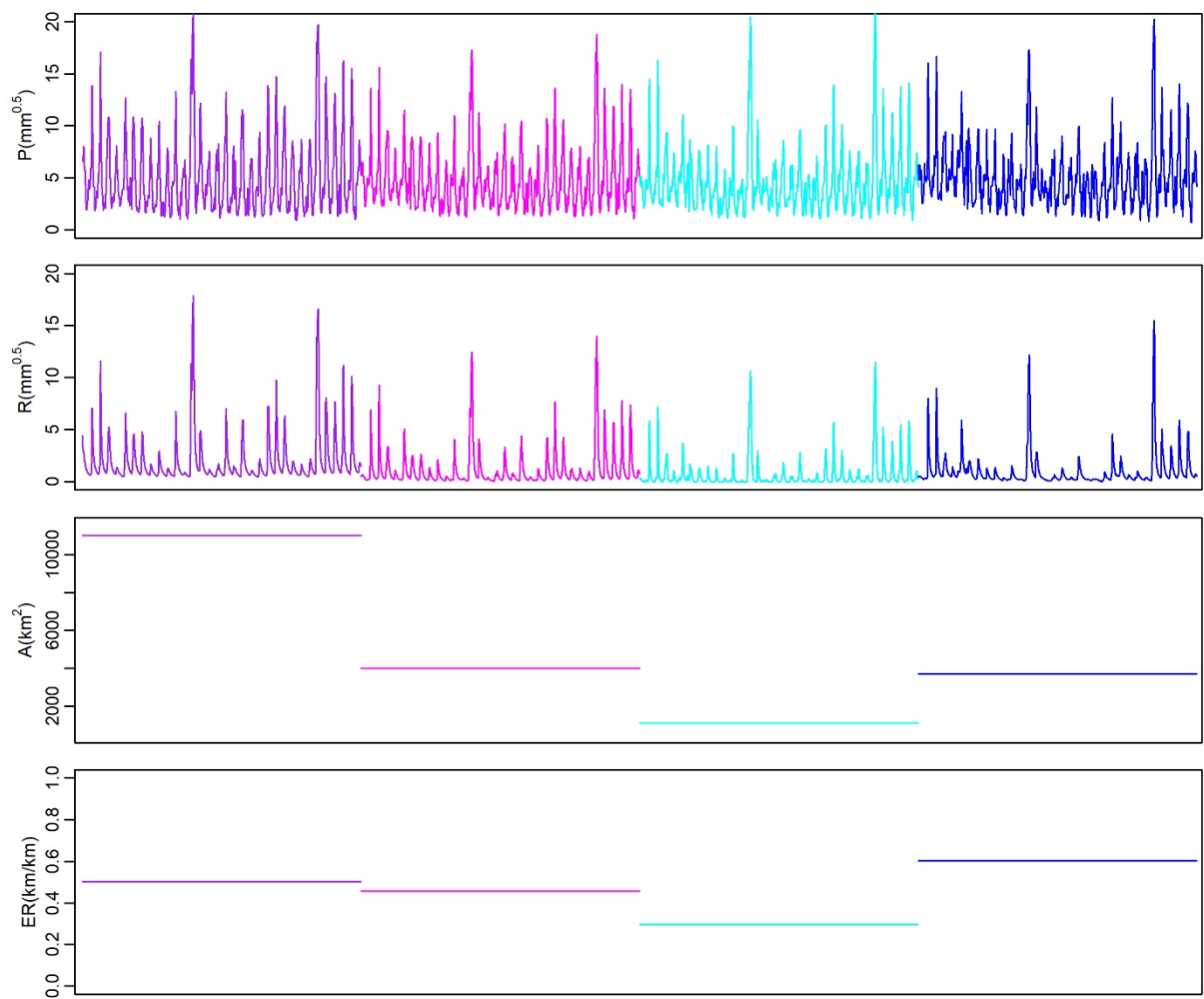
Table 4. Signs of the terms in Eq. (4) for each basin: terms in R^2P and p_e (2nd row); R^2P and p_a (3rd row); R^2P and $p_e p_a$ (4th row); R^3 and p_e (5th row); R^3 and p_a (6th row); R^3 and $p_e p_a$ (7th row).

G_{II}	B1, B4, B7, B9, B10, B11, B14, B15, B17, B18, B19, B22, B25, B29, B 30, B38, B39, B40, B44, B46, B47, B48	B8, B16, B20, B23, B26, B31, B32, B37, B42	B2, B5, B6, B12, B13, B21, B24, B28, B33, B34, B36, B41, B43, B45
$-R^2P(\beta_4$	+	—	—
$-\beta_5p_e$			
$+\beta_6p_e^2)$			
$-R^2P(-\beta_7p_a + \beta_9p_a^2)$	+	+	+
$-R^2P(\beta_8p_ep_a)$	—	—	—
$R^3(\beta_{10} - \beta_{11}p_e$	—	—	+
$+\beta_{12}p_e^2)$			
$R^3(-\beta_{13}p_a + \beta_{15}p_a^2)$	—	—	—
$R^3(\beta_{14}p_ep_a)$	+	+	+

862 FIGURES

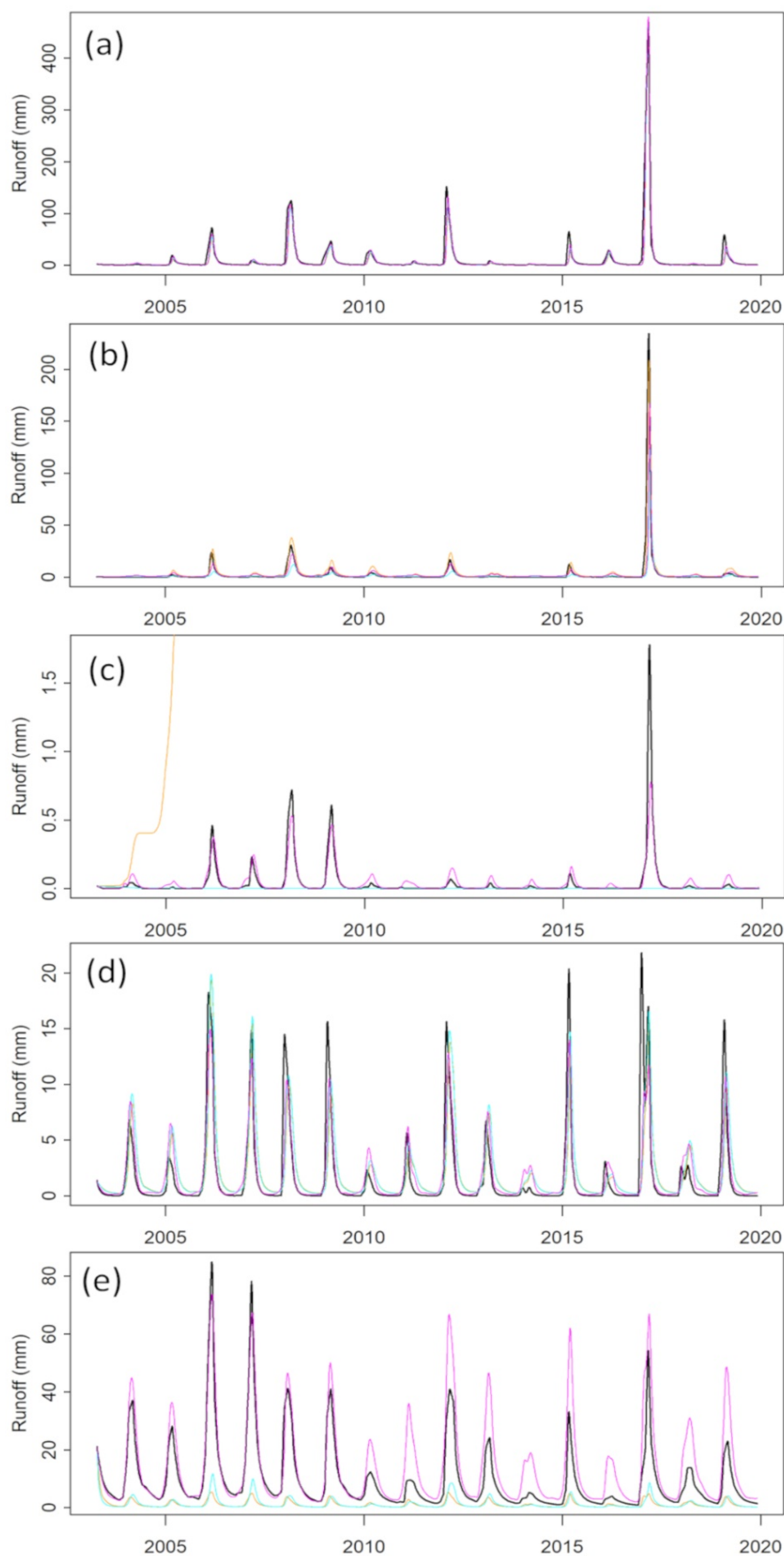


863
864 Figure 1. Region of study with the forty-nine coastal basins B01 to B49 in Peru.

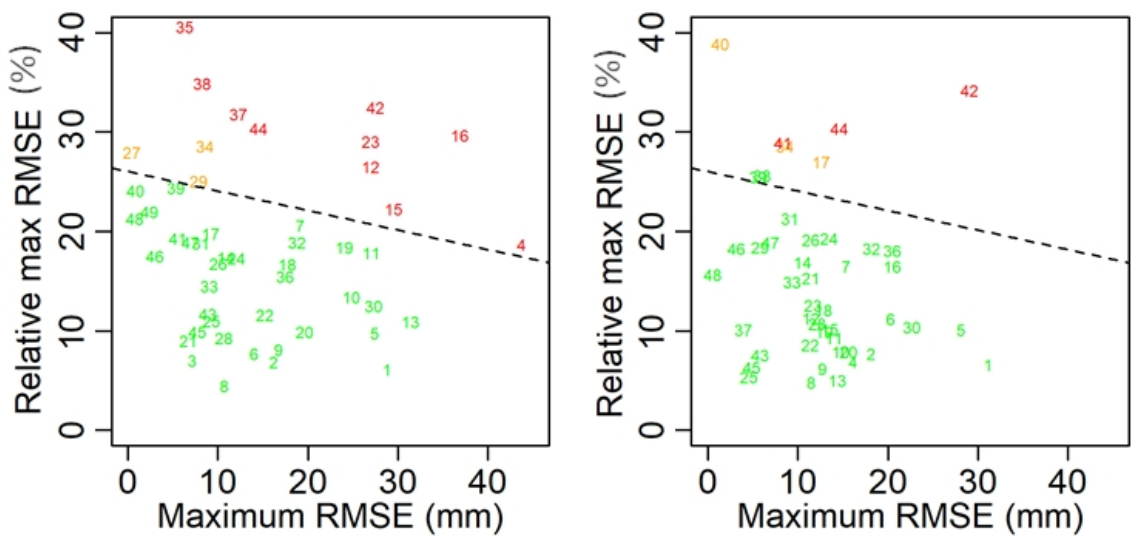


865

866 *Figure 2. Sketch of the dataset used for obtaining generalized models involving – at the same time – both*
867 *time series ($P(t)$ and $R(t)$, first two panels) and parameters (p_a and p_e , last two panels) from several*
868 *basins considered simultaneously. In this example, time series taken from basins B01 to B04 (in purple,*
869 *pink, cyan and blue colours) are concatenated in order to be processed in association (see Mangiarotti et*
870 *al. 2016). Note that during the modelling process, the discontinuities at the transition between different*
871 *basins will not be considered.*



872
873 Figure 3. Original and predicted runoff $\rho(t)$ for basins B01 (a), B04 (b), B27 (c), B39 (d) and B42 (e).
874 Original data (black), individual model (magenta), model G_I (cyan) and model G_{II} (orange).



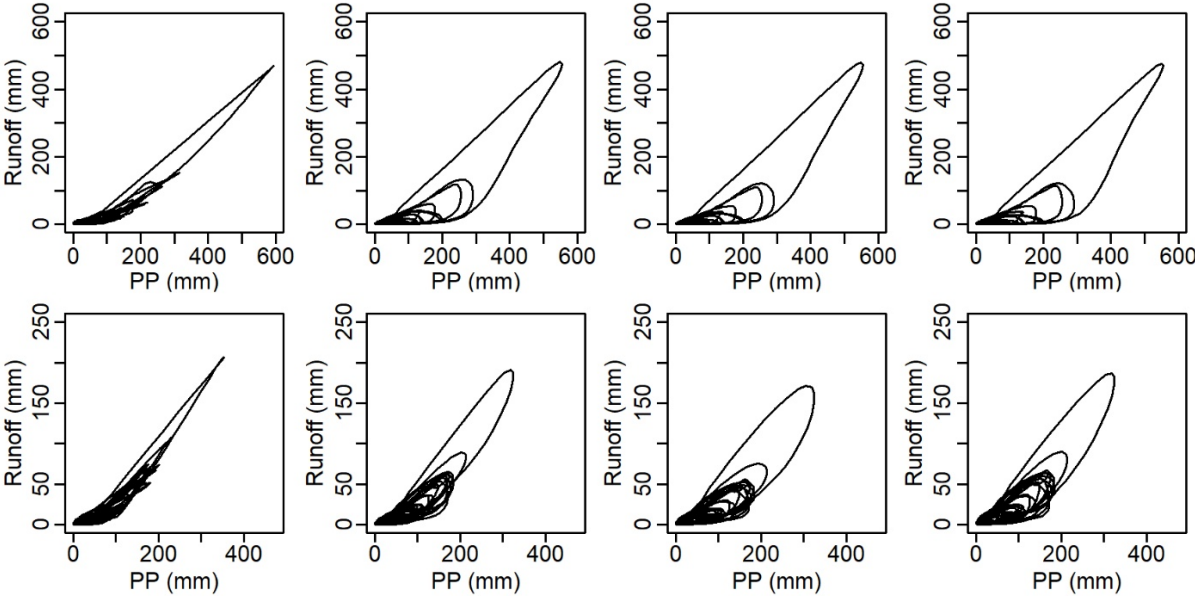
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877

878

Figure 4. Relative maximum RMSE (in %) versus maximum RMSE (in mm) for models G_I (left panel) and G_{II} (right panel). The numbers refer to the basins, and their performance is qualitatively classified (in green, orange or red) comparing the predictions with the data in the scatter plots, time series and phase portraits.

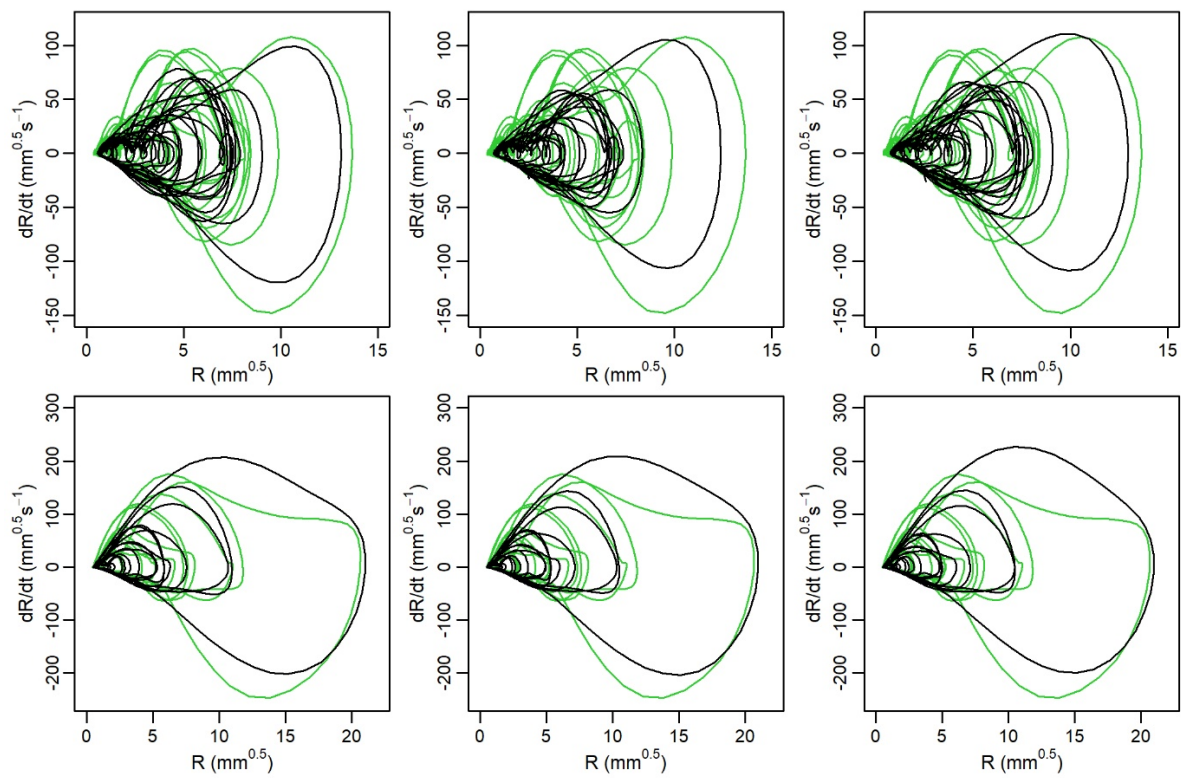


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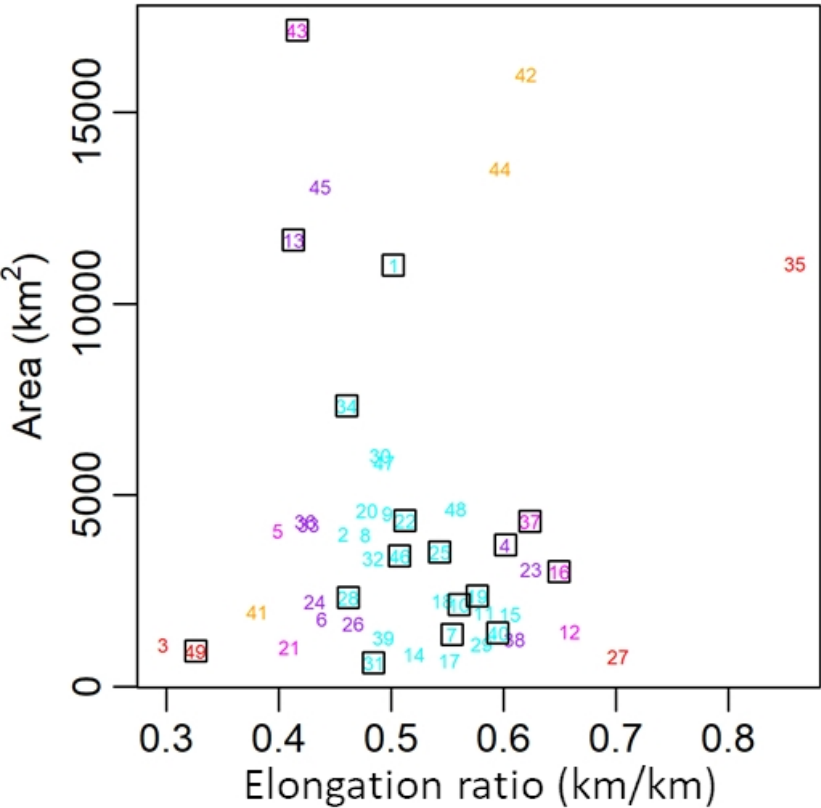
880 *Figure 5. Original and models phase portrait (ρ , τ) for basins B01 (top panels) and B09 (bottom panels),*

881 *from left to right: original data, individual model, G_I model and G_{II} model.*

882

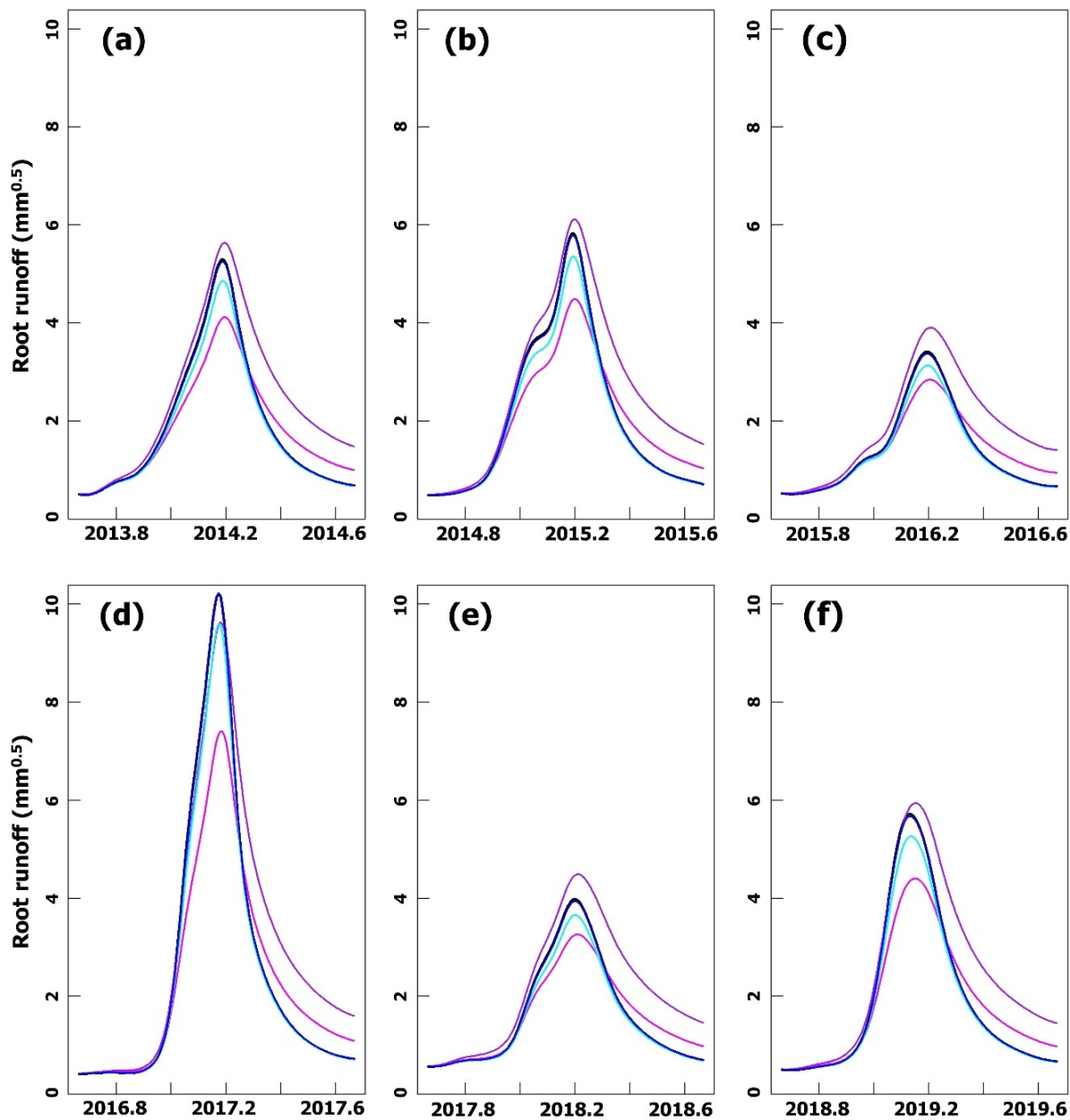


883 Figure 6. Models differential phase portrait (in black) of Runoff dynamics (R , dR/dt) for basins B01 (top
884 panels) and B09 (bottom panels), for individual model (left column), G_I model (centre column) and G_{II}
885 model (right column). Differential phase portraits reconstructed from the original time series are also
886 reported (in green) for comparison.



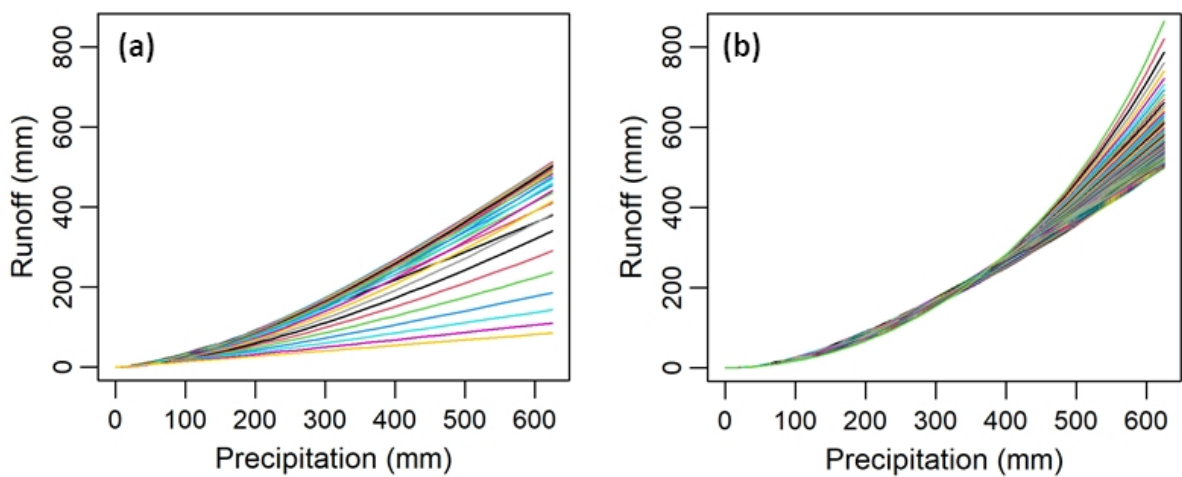
888

889 *Figure 7. Ring distribution in the parameter space (p_e, p_a) of the basin parameterization type (in red,*
890 *orange, purple and cyan) considering the contributions to the terms in RP^2 , R^2P , R^2 and R^3 . The basins*
891 *rounded by a square have been used for the identification of the model structure $\mathbf{G}_{II}$, the other ones*
892 *have been used for validation.*



893

894 Figure 8. Simulations of the root runoff $R(t)$ with model $\mathbf{G}_{II}$ under different values of p_a and p_e for six
895 hydrological years from 2013-2014 (a) to 2018-2019 (f) successively with average / average / low / high
896 / low / average precipitation. Four simulations are presented in each case, first fixing the intermediate
897 area value to $p_a = 4,334$ km² and considering elongation values $p_e = 0.4$ km/km (in purple) for less
898 rounded to $p_e = 0.65$ km/km (in magenta) for more rounded; Then by fixing an intermediate elongation
899 value to $p_e = 0.51$ km/km and considering minimum $p_a = 600$ km² (in cyan) and maximum $p_a = 7,000$
900 km² (in blue) area values.

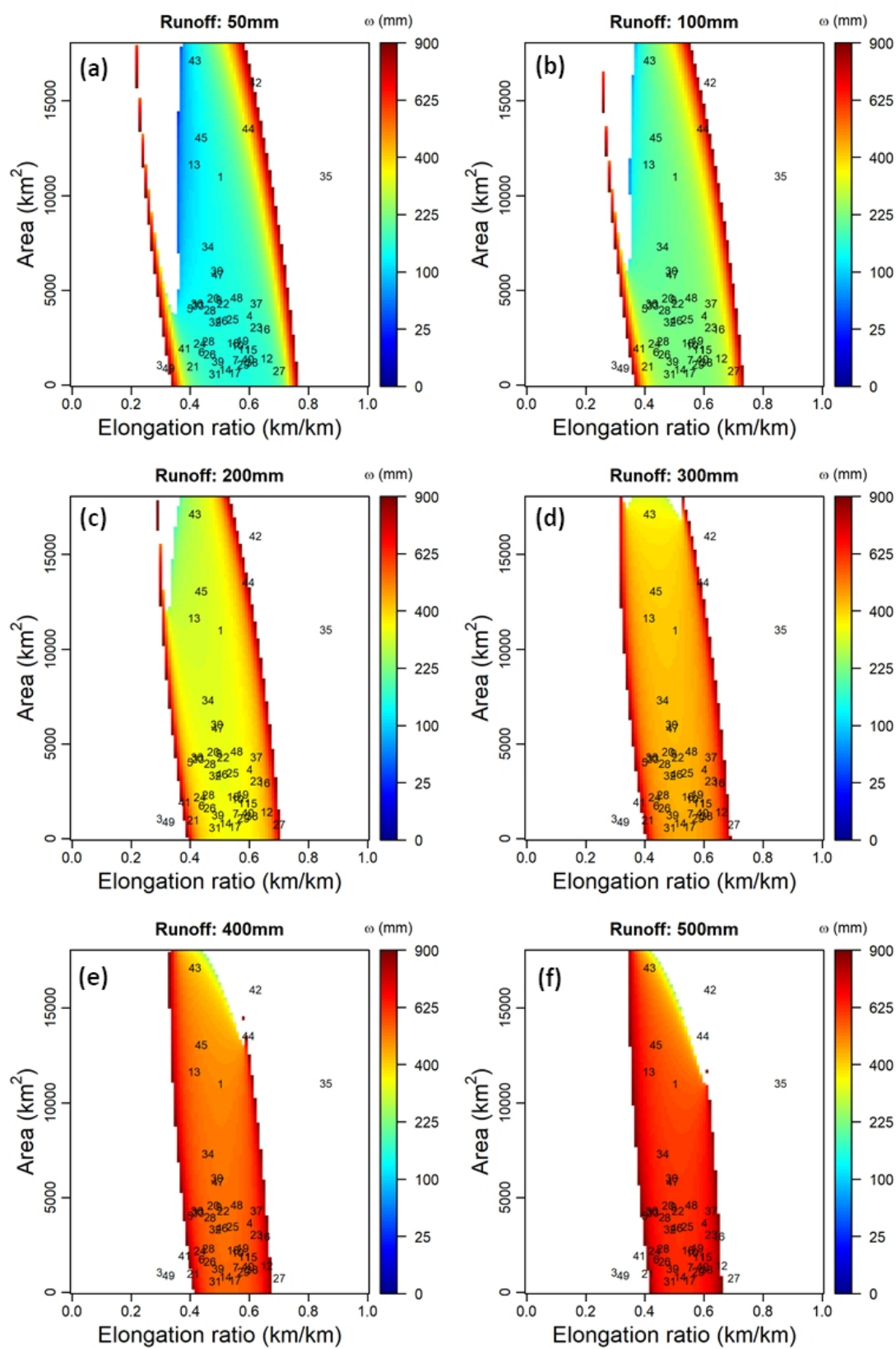


901

902 *Figure 9. Dynamical stability condition for fixed average area ($p_a = 4,334 \text{ km}^2$) with elongation ratios p_e*

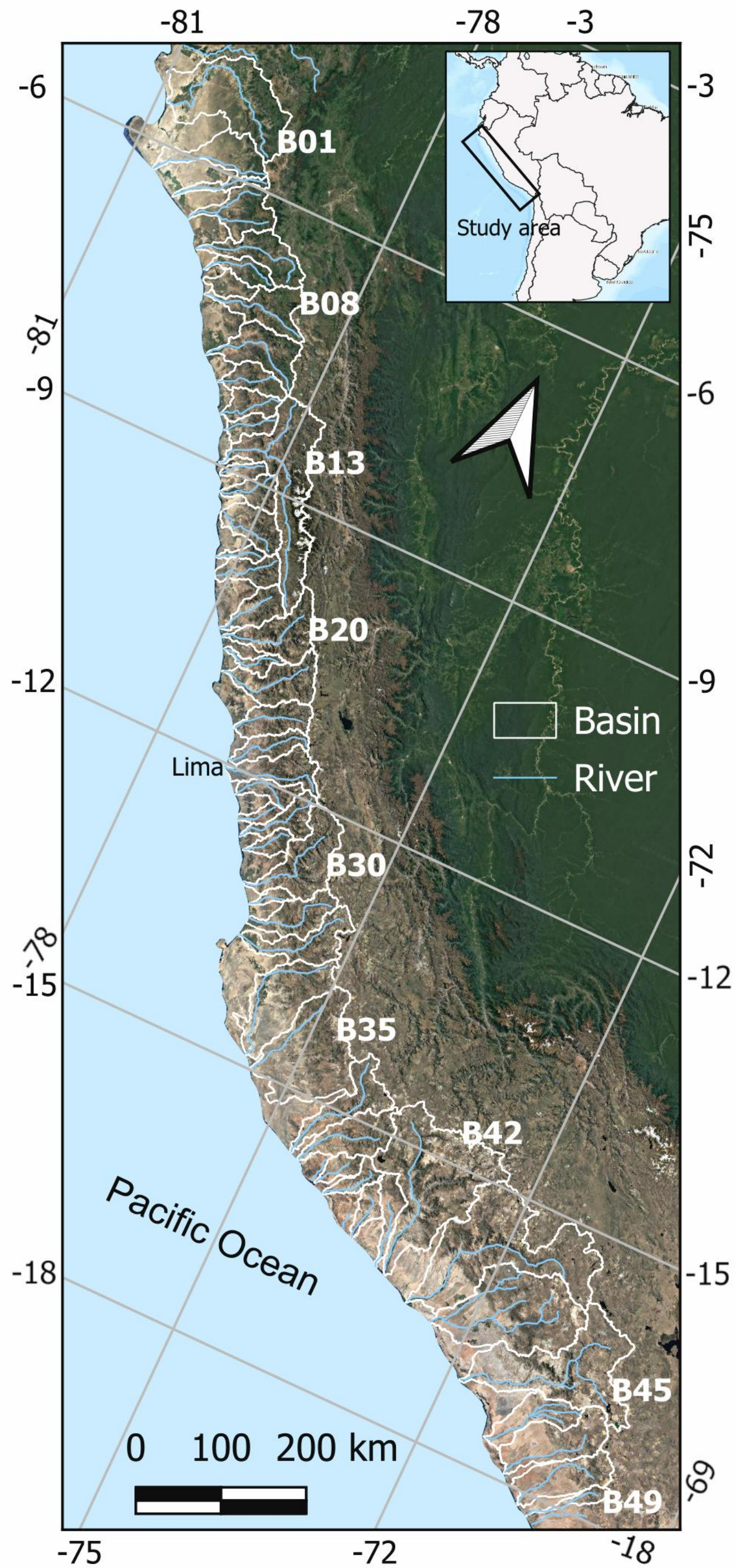
903 *varying from 0 to 1 km/km (a); And for fixed average elongation ratio ($p_e = 0.51 \text{ km/km}$) with areas p_a*

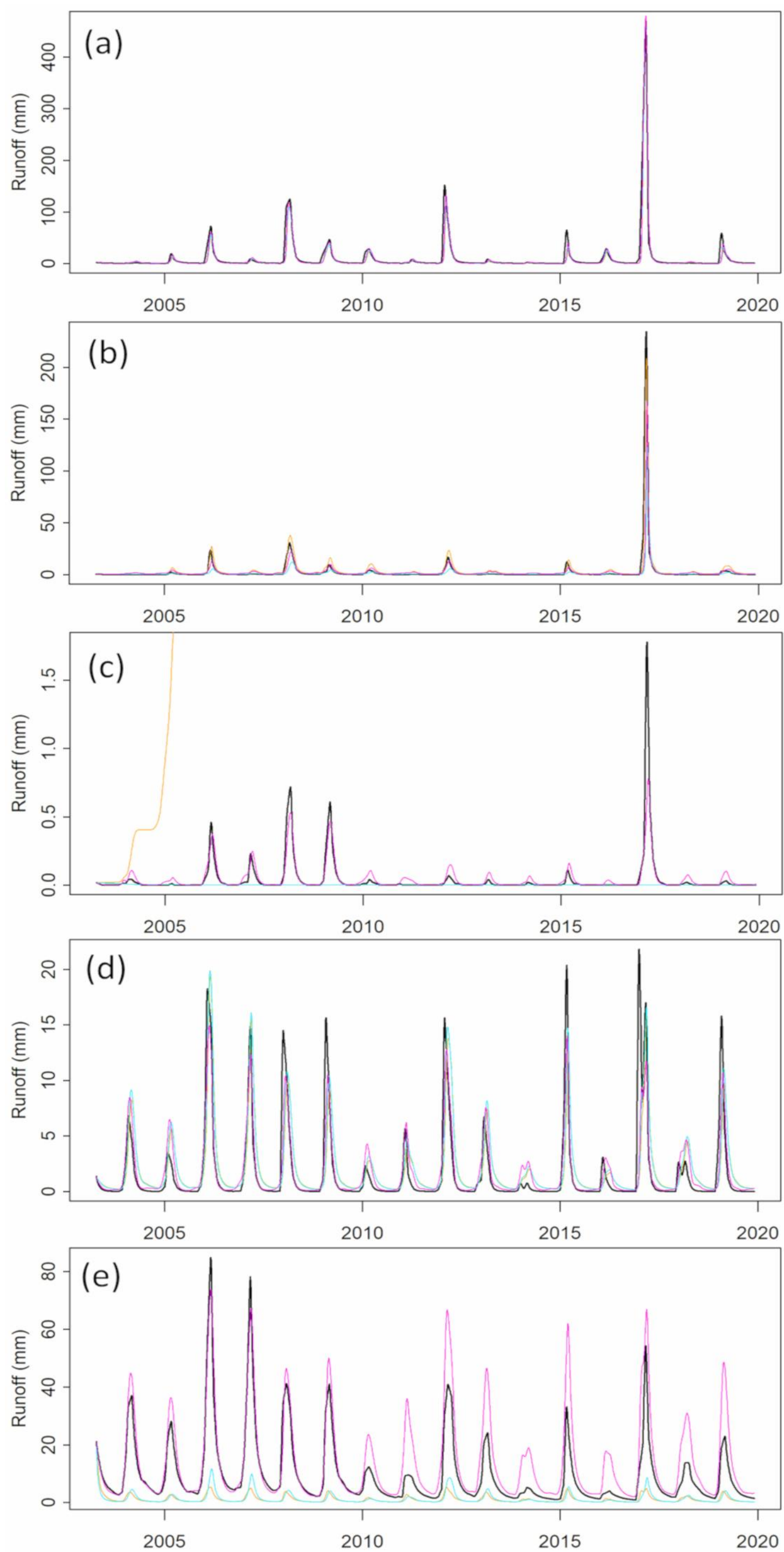
904 *varying from 0 to 18,000 km^2 (b).*

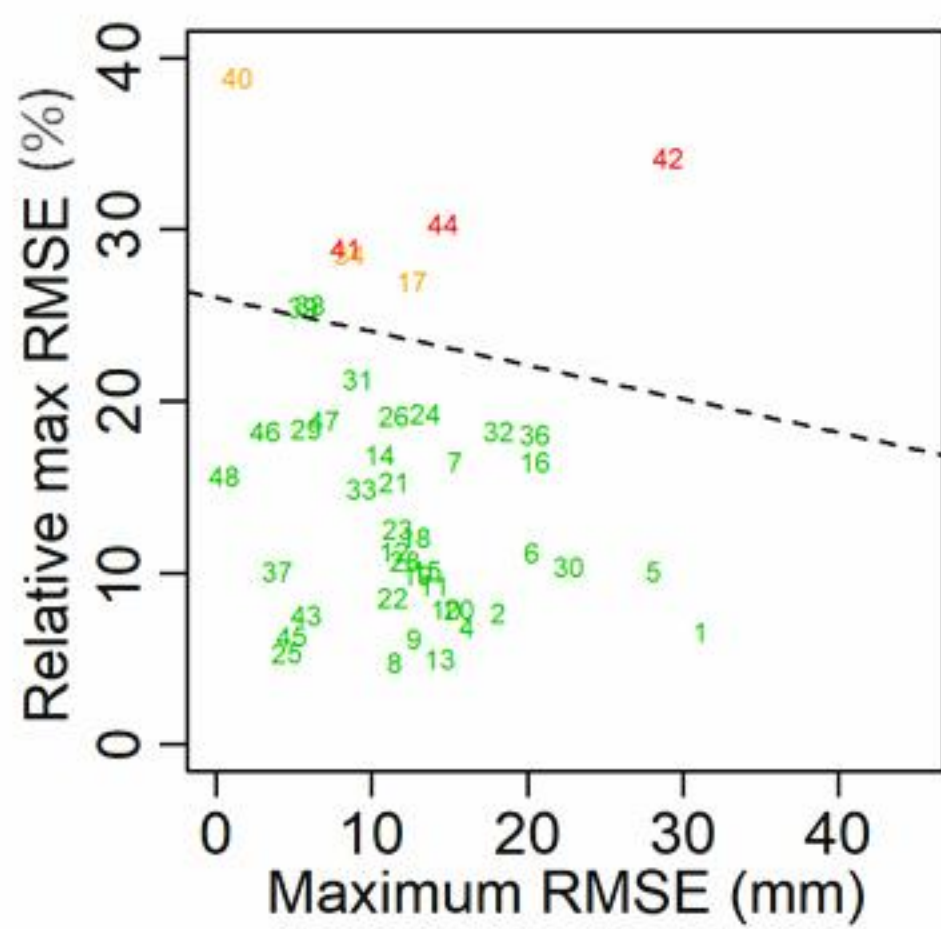
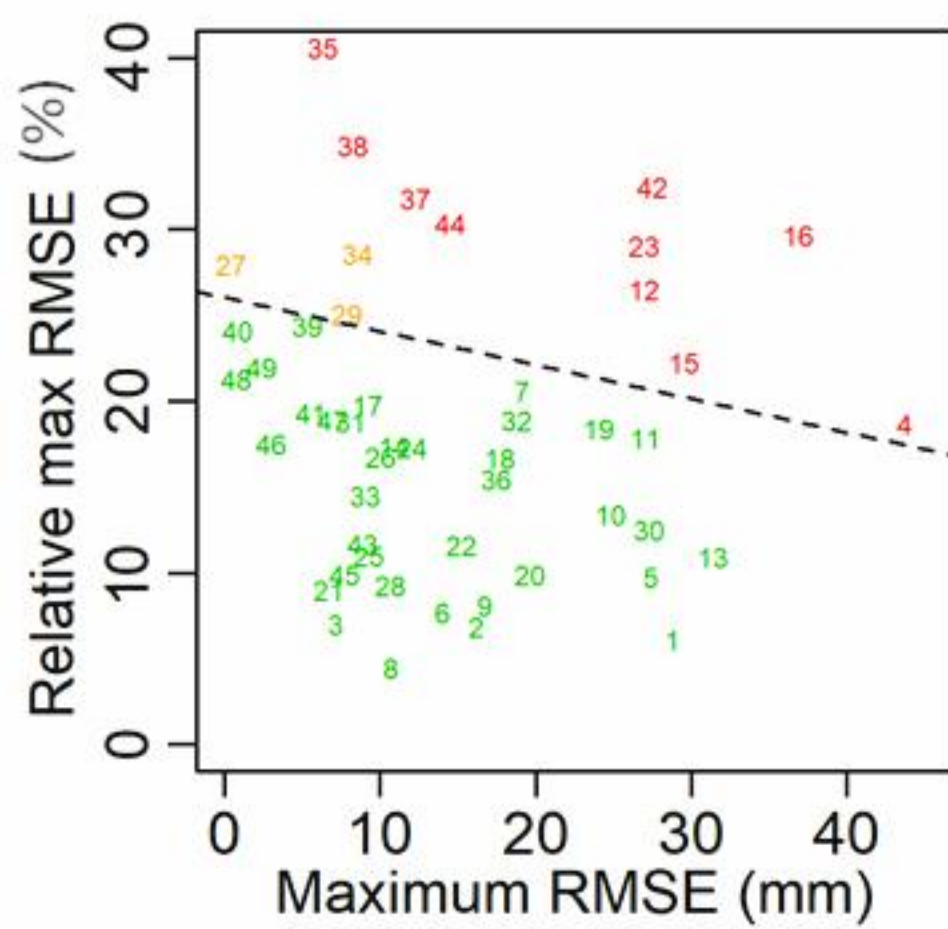


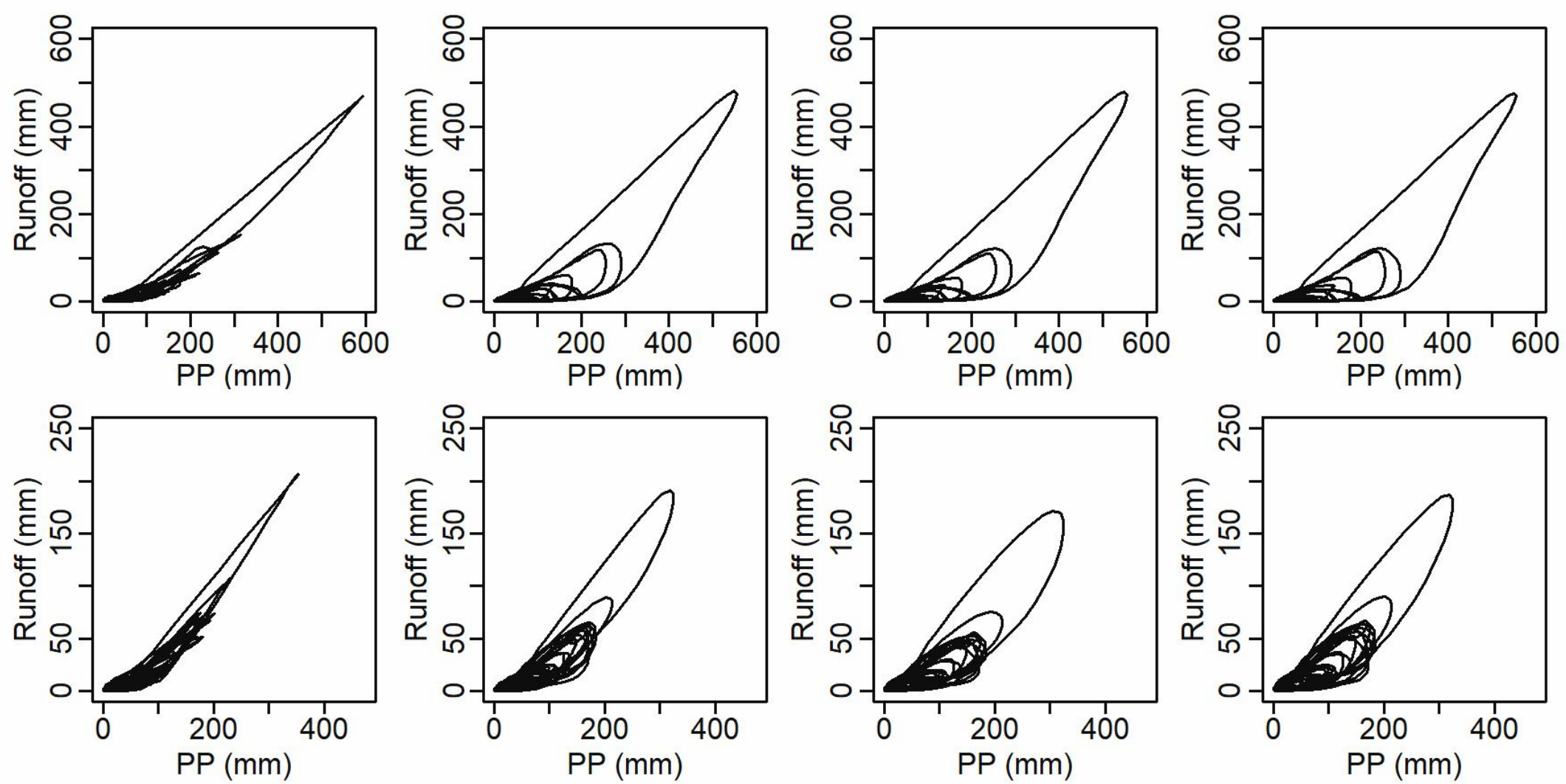
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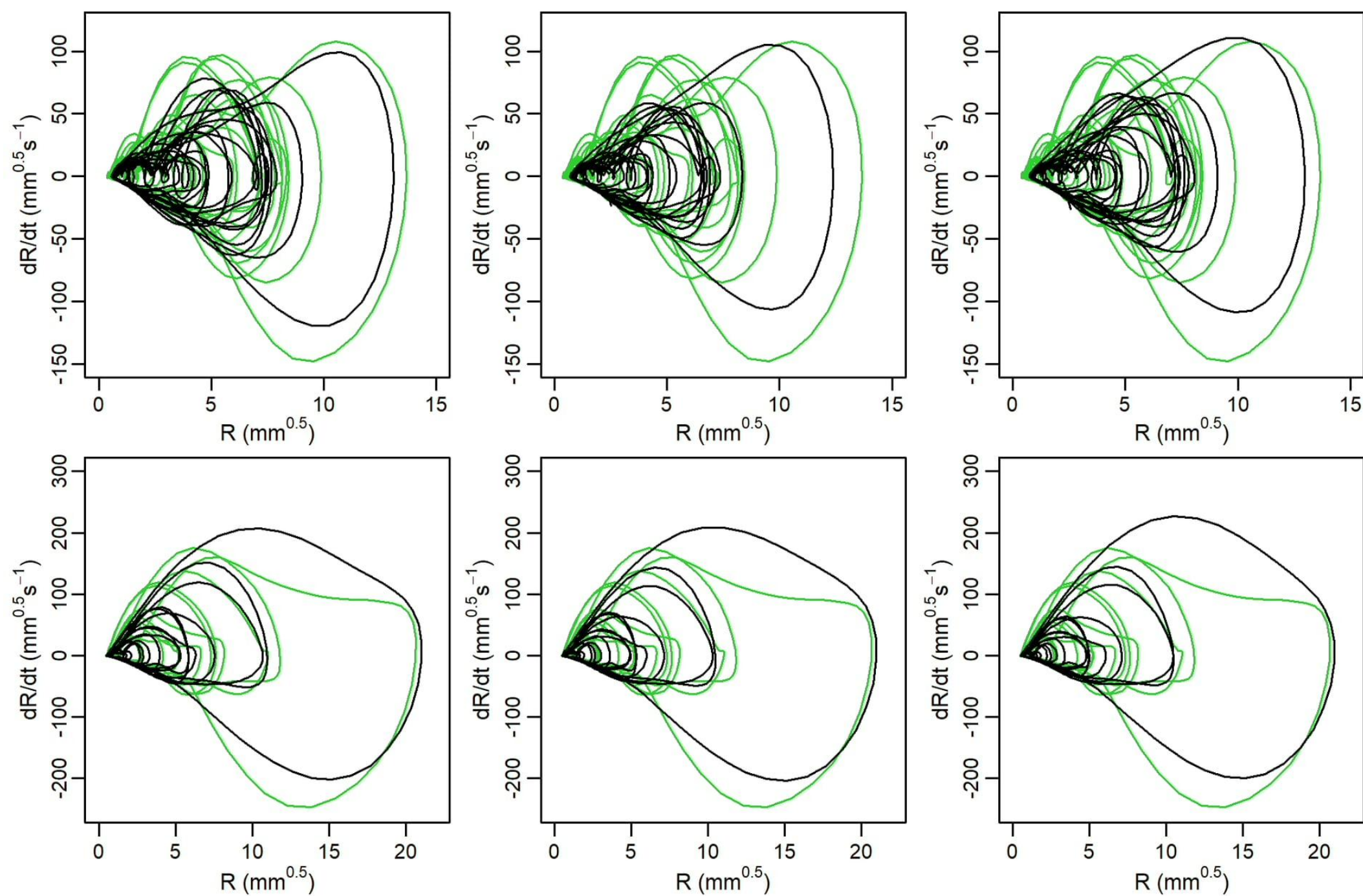
906 Figure 10. Basin stability condition in precipitation ϖ (from 0mm in blue to 900mm in red) for different
907 combinations of area and elongation ratio, to get a certain level of runoff ρ from 50mm (a) to 500mm (f).

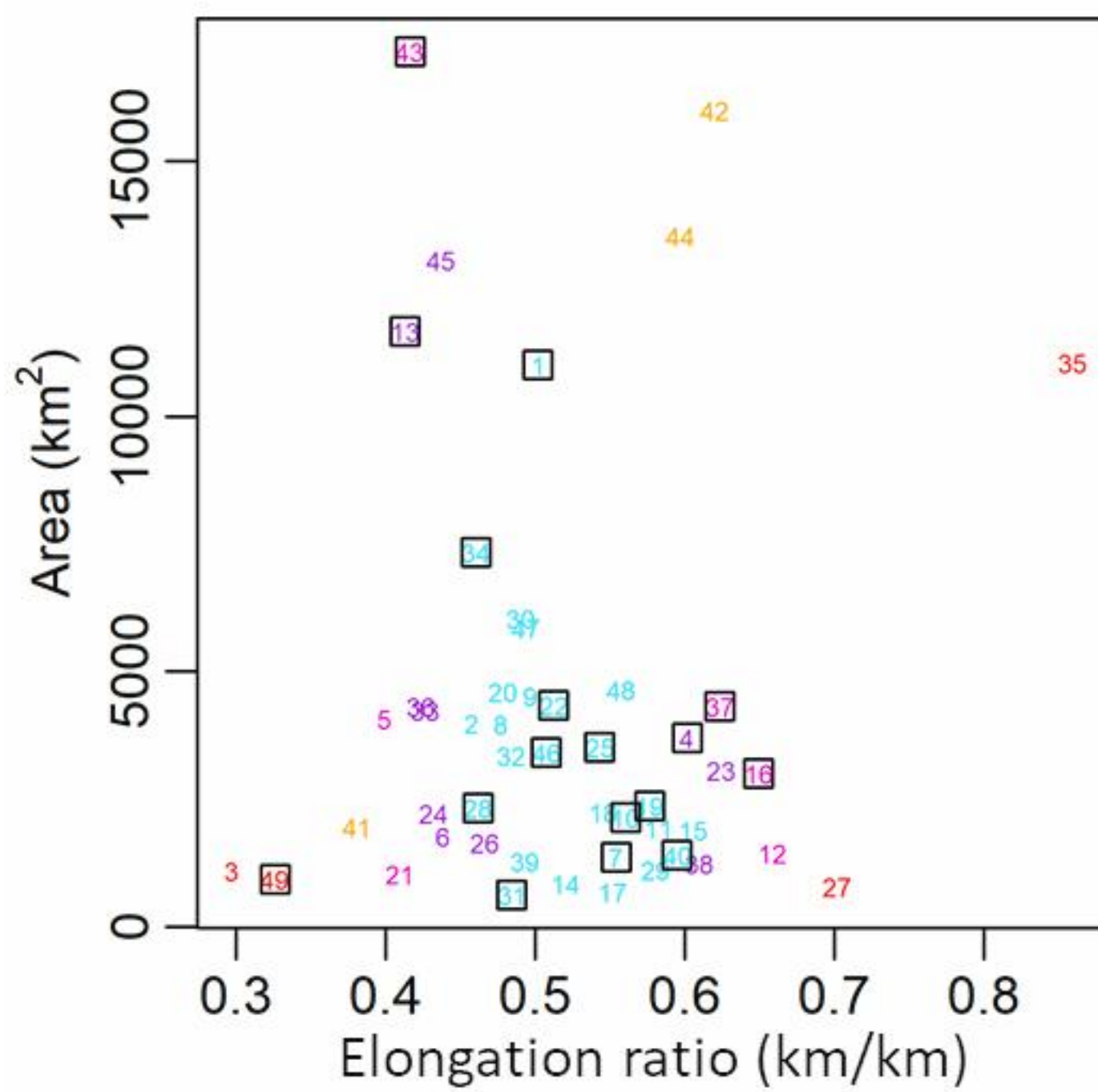


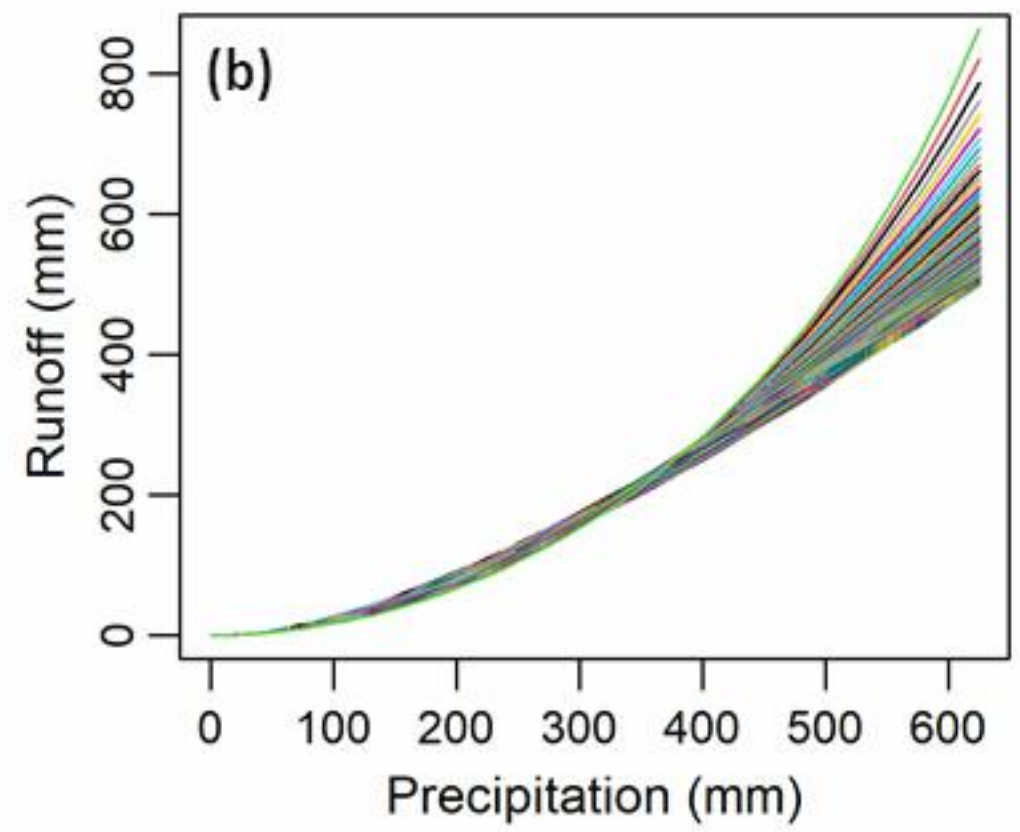
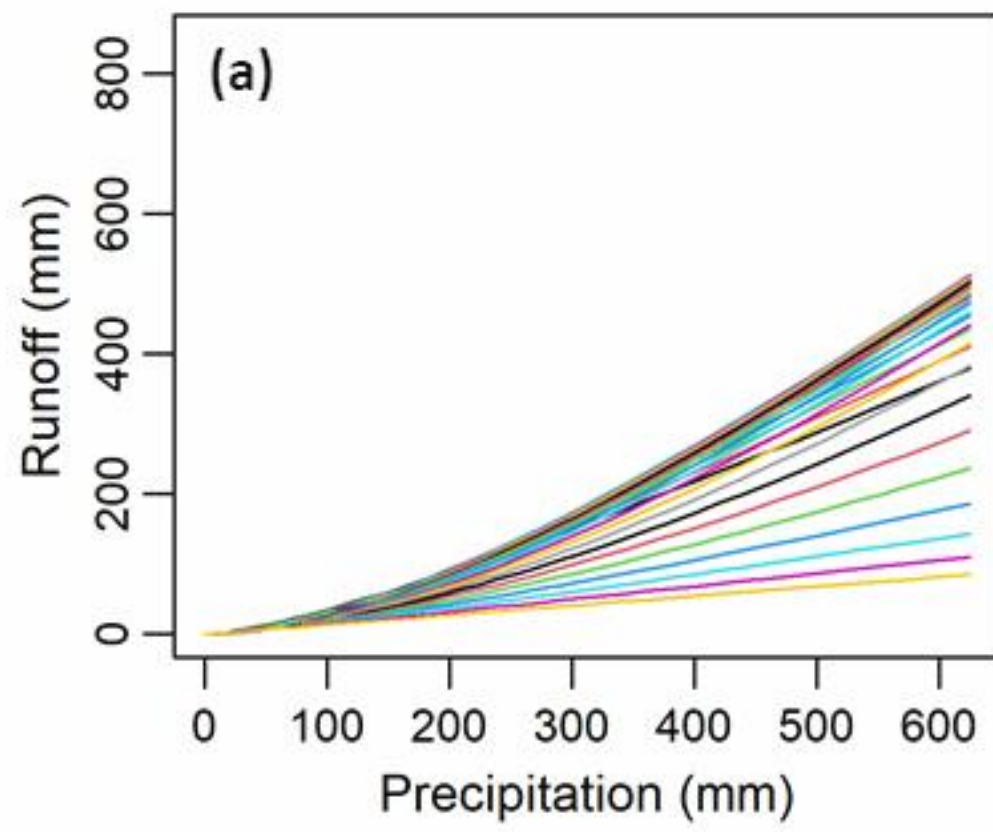


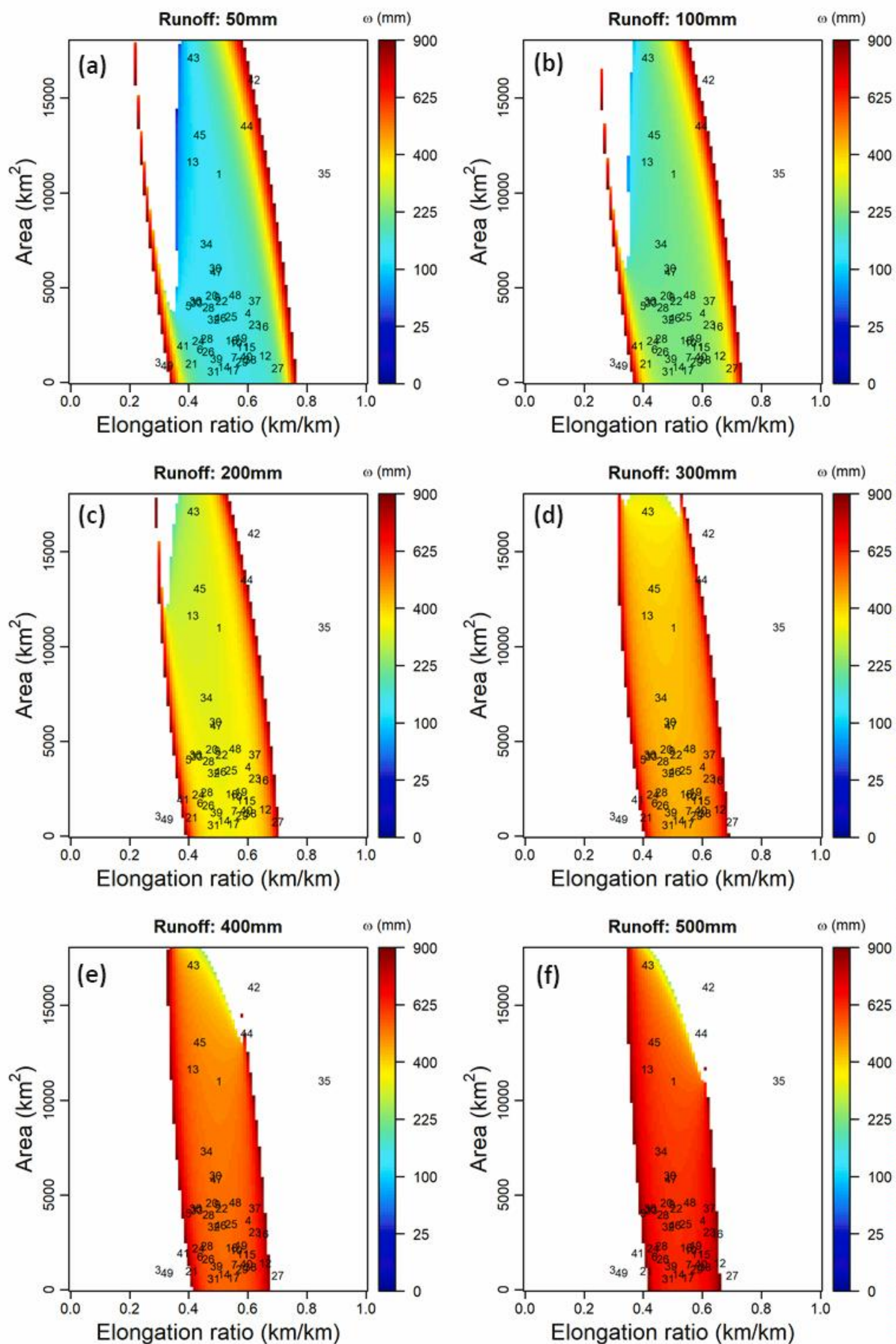












Supplementary material to:
A new class of monthly rainfall-runoff models for ungauged basins – Application to the Peruvian coastal basins
by Saéz Marina^{a,b}, Mangiarotti Sylvain^{b,*}, Rau Pedro^c, Polanco-Martínez Josué^d, Lavado-Casimiro Waldo^e, Brodu Nicolas^a, and Bourrel Luc^f

This supplementary material includes six Tables and six Figures.

TABLES

12 Table S1. Monthly rainfall (mm) and runoff (mm) statistics for each basin including the values of the minimum, first quartile
13 (1Q), median, third quartile (3Q), maximum, mean and standard deviation (SD).
14

	Rainfall						Runoff							
	Min	1Q	Median	3Q	Max	Mean	SD	Min	1Q	Median	3Q	Max	Mean	SD
B01	0.0	5.0	14.4	41.7	594.8	37.8	61.1	0.2	0.6	1.2	4.2	469.8	10.2	33.9
B02	0.0	6.3	15.6	33.6	415.5	30.7	44.4	0.0	0.1	0.2	1.3	236.1	4.7	17.8
B03	0.0	6.1	14.6	28.7	469.1	28.3	46.7	0.0	0.0	0.0	0.3	132.0	2.3	10.5
B04	0.0	8.7	19.6	36.2	428.9	31.9	43.7	0.0	0.1	0.2	0.9	239.8	3.5	17.3
B05	0.1	22.7	46.4	74.3	451.9	58.5	52.5	0.1	1.8	6.2	16.1	279.1	16.1	27.7
B06	0.1	12.7	28.0	52.4	390.6	41.8	44.9	0.0	0.3	1.3	6.0	197.7	7.7	17.9
B07	0.0	6.0	14.7	28.3	248.8	23.0	28.0	0.0	0.0	0.1	0.8	110.4	2.1	8.3
B08	0.1	15.2	45.0	82.0	431.9	62.6	64.9	0.1	0.7	2.4	11.0	244.0	13.3	28.0
B09	0.1	10.4	34.4	64.9	353.3	48.2	50.7	0.1	0.6	2.0	8.4	206.9	10.3	22.1
B10	0.1	13.0	37.9	72.9	344.0	50.4	47.1	0.0	0.7	3.0	12.4	185.6	11.1	19.0
B11	0.1	11.9	36.4	71.1	302.8	48.6	45.0	0.0	0.6	2.6	10.5	151.4	9.5	16.5
B12	0.1	10.9	33.2	63.9	283.6	44.8	42.4	0.0	0.2	0.7	3.5	101.6	4.4	9.8
B13	0.2	13.9	53.2	111.6	427.7	72.8	70.0	0.9	4.6	13.8	43.5	287.3	31.6	40.8
B14	0.1	5.8	19.5	39.7	187.2	27.3	27.1	0.0	0.1	0.4	2.8	62.5	3.1	6.4
B15	0.1	7.5	29.1	64.2	284.0	42.3	42.9	0.0	0.3	1.3	6.9	132.8	7.0	13.5
B16	0.1	4.0	19.8	52.1	251.4	33.7	38.1	0.0	0.4	1.2	5.2	124.3	5.8	12.2
B17	0.1	3.2	17.1	44.1	198.4	29.3	33.8	0.0	0.0	0.1	0.8	46.6	1.5	4.3
B18	0.1	4.4	23.2	55.3	225.0	35.6	38.1	0.0	0.3	1.2	6.7	106.0	6.6	12.8
B19	0.1	3.1	16.7	46.3	283.3	31.0	38.6	0.0	0.1	0.5	2.6	149.1	4.4	12.6
B20	0.1	6.0	31.4	82.6	335.4	52.7	57.5	0.1	0.7	3.1	18.1	201.7	15.6	27.1
B21	0.1	2.7	14.9	44.8	316.0	30.8	39.6	0.0	0.0	0.2	2.0	108.1	3.2	8.8
B22	0.1	4.7	26.1	72.9	269.7	44.5	47.9	0.1	0.5	2.3	13.0	141.9	11.0	18.5
B23	0.1	4.1	16.2	54.4	249.4	34.5	40.4	0.0	0.3	1.1	5.3	112.9	5.9	12.0
B24	0.1	3.3	15.1	51.5	205.6	31.5	36.6	0.0	0.1	0.8	6.5	86.6	6.2	11.7
B25	0.1	4.7	19.5	64.7	242.8	39.0	43.6	0.0	0.3	1.1	6.0	96.2	6.1	11.8
B26	0.1	1.9	9.1	35.1	171.6	22.5	28.3	0.0	0.0	0.2	2.3	59.9	2.9	6.5
B27	0.1	1.4	5.8	20.7	101.4	13.5	16.4	0.0	0.0	0.0	0.0	2.6	0.1	0.2
B28	0.2	2.7	13.3	51.1	252.6	32.5	40.7	0.0	0.1	0.5	5.2	114.2	5.6	12.0
B29	0.2	2.2	8.8	32.1	143.5	20.4	24.6	0.0	0.0	0.1	1.1	33.4	1.4	3.6
B30	0.3	4.7	18.6	69.8	324.8	44.2	53.1	0.1	0.9	3.2	19.0	218.6	16.5	27.3
B31	0.2	1.8	6.0	27.6	167.1	18.7	25.4	0.0	0.0	0.1	1.0	43.0	1.7	4.2
B32	0.2	4.1	13.2	46.5	207.8	30.2	36.3	0.0	0.2	0.8	5.1	99.5	6.0	12.1
B33	0.2	5.8	18.6	58.2	227.9	36.5	40.0	0.0	0.2	0.9	6.0	92.8	6.1	11.7
B34	0.1	1.7	5.6	26.4	144.3	17.9	24.2	0.0	0.2	0.4	1.5	40.7	2.1	4.7
B35	0.1	2.9	8.9	36.6	177.8	24.5	31.4	0.8	1.4	1.9	2.9	15.6	2.5	1.8
B36	0.1	4.2	14.5	63.5	323.8	41.5	53.7	0.0	0.1	0.7	6.7	112.8	7.9	15.8
B37	0.1	3.1	11.5	52.4	247.6	33.3	43.4	0.0	0.2	0.7	3.1	65.3	3.6	7.4
B38	0.1	2.9	10.1	39.2	171.5	25.7	31.7	0.0	0.0	0.2	2.0	48.2	2.5	5.5
B39	0.1	3.0	10.8	41.9	184.7	27.8	34.7	0.0	0.0	0.2	1.9	48.2	2.5	5.5
B40	0.0	1.7	6.4	25.0	117.4	17.2	22.1	0.0	0.0	0.0	0.2	9.9	0.3	0.9
B41	0.0	1.7	6.9	27.8	133.5	19.2	25.4	0.0	0.0	0.1	1.2	35.1	1.9	4.4
B42	0.1	4.9	17.5	69.2	287.3	46.5	58.5	0.9	4.5	8.4	20.5	207.4	18.1	24.5
B43	0.0	2.9	13.3	54.3	229.9	36.2	46.6	0.1	1.1	2.5	9.2	117.0	9.1	15.4
B44	0.0	1.5	8.2	37.2	172.2	24.9	33.2	0.4	1.3	2.2	5.2	69.8	5.0	7.3
B45	0.0	2.5	11.3	50.6	211.5	33.1	43.3	0.1	0.7	1.9	7.6	98.0	7.6	13.2
B46	0.0	1.2	4.6	23.0	132.9	16.6	23.7	0.0	0.0	0.1	0.5	21.8	0.9	2.1
B47	0.0	1.4	5.3	25.6	148.5	18.0	25.0	0.0	0.2	0.5	1.9	51.6	2.3	4.6
B48	0.0	1.8	5.7	19.5	100.3	14.2	18.1	0.0	0.1	0.2	0.5	12.3	0.5	1.1
B49	0.1	2.8	8.1	27.7	123.9	19.6	24.4	0.0	0.0	0.0	0.9	22.0	1.4	3.0

15 Table S2. Global runoff errors for the individual and generalized models on the validation period (2003-2023). A
16 gray background is applied when the basin is used in the modelling process.

Basin	Runoff Max (mm)	Individual		G _I		G _{II}	
		R ²	Global RMSE (mm)	R ²	Global RMSE (mm)	R ²	Global RMSE (mm)
B01	469.8	1.0	10.3	1.0	11.5	0.9	12.4
B02	236.1	1.0	4.5	0.9	5.7	0.9	6.6
B03	103.4	0.8	4.4	0.9	2.5	NA	Inf
B04	234.4	0.8	9.0	0.8	11.0	1.0	4.6
B05	279.1	1.0	5.4	1.0	13.0	0.9	12.2
B06	181.7	1.0	4.6	0.9	6.2	0.9	8.1
B07	93.0	0.9	3.0	0.7	5.3	0.8	4.3
B08	239.9	1.0	6.5	1.0	6.5	1.0	8.0
B09	206.9	1.0	4.9	0.9	7.6	1.0	6.4
B10	185.6	1.0	4.2	0.9	11.7	0.9	7.2
B11	151.4	1.0	3.7	0.8	12.0	0.9	6.3
B12	101.6	0.9	4.1	0.1	9.5	0.9	6.1
B13	287.3	1.0	8.1	1.0	19.6	1.0	7.9
B14	62.5	1.0	2.1	0.8	4.3	0.8	4.1
B15	132.8	1.0	4.1	0.7	13.5	0.9	6.4
B16	124.3	1.0	4.9	0.4	16.4	0.9	8.7
B17	46.6	0.9	2.4	0.9	3.7	0.9	5.0
B18	106.0	1.0	2.9	0.9	8.5	0.9	6.2
B19	130.2	1.0	3.3	0.8	6.9	0.9	3.8
B20	198.7	1.0	4.6	1.0	8.6	1.0	6.8
B21	74.8	1.0	1.4	0.9	3.0	0.8	4.3
B22	131.7	1.0	4.3	1.0	7.1	1.0	5.2
B23	92.7	1.0	2.8	0.7	10.3	0.9	4.2
B24	69.7	0.9	3.2	0.9	5.0	0.8	5.8
B25	84.7	1.0	2.5	1.0	3.5	1.0	2.2
B26	59.9	0.9	2.4	0.9	3.7	0.8	4.3
B27	1.8	0.9	0.1	-0.1	0.2	NA	Inf
B28	114.2	1.0	2.8	0.9	5.1	0.9	5.9
B29	31.5	1.0	0.9	0.6	3.1	0.8	2.3
B30	218.6	1.0	5.2	1.0	14.5	0.9	12.2
B31	43.0	0.9	2.1	0.8	2.9	0.7	3.2
B32	99.5	1.0	3.0	0.9	7.1	0.9	7.1
B33	62.7	0.9	4.2	1.0	3.8	0.9	4.7
B34	30.1	0.9	1.8	0.8	2.9	0.7	3.0
B35	15.6	0.9	0.5	0.0	2.6	NA	Inf
B36	112.8	0.9	6.1	0.9	5.8	0.9	8.3
B37	38.7	0.9	2.7	0.8	5.5	1.0	2.2
B38	23.7	0.9	1.9	0.5	3.8	0.7	2.6
B39	21.8	0.9	1.8	0.8	2.5	0.7	2.6
B40	3.7	0.9	0.2	0.5	0.3	0.6	0.8
B41	29.1	0.9	1.6	0.7	2.1	0.3	5.2
B42	84.8	0.9	7.8	0.8	13.6	0.7	14.1
B43	76.0	1.0	3.0	1.0	4.3	1.0	3.1
B44	48.0	0.9	2.8	0.8	6.1	0.9	5.9
B45	77.9	0.9	3.2	1.0	3.4	1.0	2.7
B46	17.3	0.8	1.0	0.8	1.1	0.7	1.1
B47	36.9	0.9	1.4	0.8	2.5	0.8	2.6
B48	3.6	1.0	0.2	0.8	0.3	0.9	0.3
B49	11.1	1.0	0.6	0.7	1.4	NA	Inf

19 Table S3. Statistics of the RMSEs per cycle for each model on the validation period (2003-2023): mean and standard deviation (SD), 90% quantile (Q90), maximum (Max) and
 20 relative (Rel) maximum (RMSE maximum/runoff maximum). A gray background is applied when the basin is used in the modelling process.
 21

		Indiv. model					G _I model					G _{II} model				
Runoff		RMSEs per cycle					RMSEs per cycle					RMSEs per cycle				
Basin	Max	Mean	SD	Q90	Max	Rel	Mean	SD	Q90	Max	Rel	Mean	SD	Q90	Max	Rel
B01	469.8	6.8	8.3	19.6	25.9	0.06	7.6	9.2	21.7	28.9	0.06	8.2	9.9	23.3	31.2	0.07
B02	236.1	3.1	3.5	8.0	11.5	0.05	4.0	4.4	9.2	16.2	0.07	4.7	5.0	11.0	18.1	0.08
B03	103.4	2.2	4.2	3.3	16.9	0.16	2.0	1.7	3.1	7.2	0.07	Inf	Inf	0.0	Inf	Inf
B04	234.4	3.9	8.9	3.9	35.9	0.15	4.5	11.1	6.5	43.8	0.19	3.1	3.8	3.8	16.2	0.07
B05	279.1	4.8	2.4	7.9	8.9	0.03	11.1	7.4	19.7	27.5	0.10	10.7	6.4	16.7	28.2	0.10
B06	181.7	4.0	2.1	6.2	8.7	0.05	5.1	3.5	8.4	14.0	0.08	6.6	5.0	10.9	20.4	0.11
B07	93.0	1.9	2.6	3.7	10.3	0.11	3.0	4.8	5.7	19.2	0.21	2.5	3.8	4.7	15.4	0.17
B08	239.9	5.9	3.0	10.0	10.5	0.04	6.0	2.7	9.9	10.8	0.04	7.6	2.2	10.9	11.6	0.05
B09	206.9	4.4	2.4	7.7	8.3	0.04	6.4	4.5	10.9	16.8	0.08	5.6	3.2	9.0	12.7	0.06
B10	185.6	3.9	1.9	6.7	8.2	0.04	10.9	5.3	14.9	24.8	0.13	6.7	3.3	10.2	14.7	0.08
B11	151.4	3.5	1.7	5.9	6.6	0.04	11.0	6.0	15.2	27.1	0.18	5.8	3.1	8.4	14.0	0.09
B12	101.6	3.3	2.7	5.4	11.3	0.11	7.6	6.6	12.8	26.9	0.27	6.0	1.6	6.5	11.5	0.11
B13	287.3	7.9	2.9	11.7	14.8	0.05	19.0	7.6	28.2	31.4	0.11	7.7	2.9	11.4	14.4	0.05
B14	62.5	2.0	0.9	3.5	3.7	0.06	3.6	2.8	6.3	10.9	0.17	3.5	2.6	6.0	10.5	0.17
B15	132.8	3.6	2.5	6.4	10.4	0.08	11.9	8.1	21.8	29.5	0.22	5.6	3.7	11.4	13.5	0.10
B16	124.3	4.2	3.2	8.6	10.4	0.08	13.2	11.5	31.1	36.9	0.30	6.8	6.3	16.3	20.5	0.16
B17	46.6	2.0	1.7	4.3	5.5	0.12	3.0	2.6	6.4	9.2	0.20	4.1	3.3	7.8	12.6	0.27
B18	106.0	2.7	1.6	5.0	6.5	0.06	7.4	5.1	15.0	17.7	0.17	5.5	3.6	10.7	12.8	0.12
B19	130.2	2.2	2.7	3.8	11.2	0.09	4.2	6.1	8.6	24.0	0.18	2.7	3.0	4.4	12.9	0.10
B20	198.7	4.3	2.1	5.9	10.8	0.05	7.7	4.6	11.8	19.6	0.10	6.2	3.5	8.8	15.6	0.08
B21	74.8	1.2	1.0	2.0	4.0	0.05	2.8	1.5	4.2	6.8	0.09	3.8	2.5	5.8	11.4	0.15
B22	131.7	3.9	2.2	5.9	8.6	0.07	6.2	4.0	10.0	15.2	0.12	4.6	2.8	7.0	11.4	0.09
B23	92.7	2.3	1.9	4.3	7.5	0.08	8.2	7.3	16.6	27.0	0.29	3.2	3.0	6.4	11.7	0.13
B24	69.7	2.8	1.5	3.5	7.7	0.11	4.4	2.8	6.9	12.1	0.17	5.1	3.1	8.1	13.5	0.19
B25	84.7	2.1	1.6	3.7	6.5	0.08	2.7	2.5	5.4	9.3	0.11	2.1	0.8	2.9	4.6	0.05

B26	59.9	2.0	1.4	2.8	6.3	0.11	3.1	2.4	5.1	10.1	0.17	3.5	2.8	6.0	11.5	0.19
B27	1.8	0.0	0.1	0.1	0.3	0.16	0.1	0.1	0.2	0.5	0.28	Inf	Inf	15.1	Inf	Inf
B28	114.2	2.6	1.2	4.4	4.8	0.04	4.7	2.6	8.4	10.6	0.09	5.4	3.0	9.7	12.2	0.11
B29	31.5	0.7	0.6	1.6	2.3	0.07	2.2	2.5	6.0	7.9	0.25	1.8	1.7	4.2	5.8	0.18
B30	218.6	4.7	2.6	6.7	12.9	0.06	13.7	6.3	22.3	27.3	0.12	11.5	5.2	18.7	22.6	0.10
B31	43.0	1.7	1.4	2.6	5.9	0.14	2.2	2.0	3.9	8.1	0.19	2.4	2.3	4.4	9.1	0.21
B32	99.5	2.3	2.2	4.2	9.4	0.09	5.5	5.0	10.9	18.8	0.19	5.5	4.9	11.0	18.2	0.18
B33	62.7	3.6	2.3	6.3	9.6	0.15	3.1	2.5	6.8	9.1	0.14	4.4	2.0	7.0	9.4	0.15
B34	30.1	1.2	1.5	3.0	5.4	0.18	1.9	2.5	5.0	8.6	0.29	2.0	2.5	5.1	8.6	0.29
B35	15.6	0.4	0.4	0.7	1.7	0.11	2.4	1.3	3.3	6.3	0.41	Inf	Inf	Inf	Inf	Inf
B36	112.8	4.8	4.0	6.3	18.6	0.16	4.5	3.9	6.9	17.5	0.16	7.4	3.9	9.3	20.5	0.18
B37	38.7	2.1	1.8	5.0	5.5	0.14	4.3	3.7	8.6	12.3	0.32	2.1	0.6	2.6	3.9	0.10
B38	23.7	1.5	1.2	2.6	5.3	0.22	3.1	2.4	5.6	8.3	0.35	2.3	1.4	3.5	6.1	0.26
B39	21.8	1.6	1.0	2.3	4.9	0.22	2.2	1.2	3.3	5.3	0.24	2.3	1.4	3.5	5.6	0.25
B40	3.7	0.1	0.1	0.2	0.5	0.15	0.2	0.2	0.3	0.9	0.24	0.7	0.4	1.3	1.4	0.39
B41	29.1	1.2	1.0	2.5	3.7	0.13	1.6	1.4	3.1	5.6	0.19	5.0	1.9	7.6	8.4	0.29
B42	84.8	7.2	3.2	11.5	12.0	0.14	12.1	7.7	22.0	27.5	0.32	12.4	8.2	22.8	29.0	0.34
B43	76.0	2.9	1.1	4.4	5.3	0.07	3.8	2.5	7.6	8.9	0.12	2.9	1.4	4.8	5.8	0.08
B44	48.0	2.2	1.8	4.1	7.1	0.15	5.3	3.4	8.3	14.6	0.30	5.1	3.4	8.0	14.6	0.30
B45	77.9	2.7	1.6	4.8	6.8	0.09	2.8	2.1	5.7	7.7	0.10	2.5	1.2	4.3	4.9	0.06
B46	17.3	0.7	0.8	1.6	2.9	0.17	0.8	0.7	1.6	3.0	0.17	0.9	0.8	1.6	3.2	0.18
B47	36.9	1.1	0.8	1.8	3.6	0.10	1.8	1.7	3.2	7.0	0.19	1.9	1.7	3.3	7.0	0.19
B48	3.6	0.2	0.1	0.2	0.3	0.09	0.3	0.2	0.6	0.8	0.21	0.3	0.1	0.4	0.6	0.16
B49	11.1	0.5	0.3	0.9	1.2	0.11	1.2	0.7	2.1	2.4	0.22	Inf	Inf	0.0	Inf	Inf

22

23

Table S4. Mean value for each term of Eq. (4): terms in R^2P (1st column), terms in RP^2 (2nd column), sum of terms in R^2P and RP^2 (3rd column), terms in R^2 (4th column), terms in R^3 (5th column) and sum of terms in R^2 and R^3 (6th column).

Basin	$-R^2P(\beta_4 - \beta_5p_e + \beta_6p_e^2 - \beta_7p_a + \beta_8p_ap_a + \beta_9p_a^2)$	β_0RP^2	$-R^2P(\beta_4 - \beta_5p_e + \beta_6p_e^2 - \beta_7p_a + \beta_8p_ap_a + \beta_9p_a^2)$	$R^2(\beta_1 - \beta_2p_e + \beta_3p_e^2)$	$R^3(\beta_{10} - \beta_{11}p_e + \beta_{12}p_e^2 - \beta_{13}p_a + \beta_{14}p_ep_a + \beta_{15}p_a^2)$	$R^2(\beta_1 - \beta_2p_e + \beta_3p_e^2) + R^3(\beta_{10} - \beta_{11}p_e + \beta_{12}p_e^2 - \beta_{13}p_a + \beta_{14}p_ep_a + \beta_{15}p_a^2)$
B01	3.5	30.9	34.4	-28.7	-5.7	-34.4
B02	1.9	17.2	19.1	-14.7	-4.4	-19.0
B03	NA	NA	NA	NA	NA	NA
B04	-0.3	13.8	13.5	-11.8	-1.6	-13.4
B05	-32.4	48.2	15.8	-18.8	3.1	-15.7
B06	-5.9	20.4	14.5	-14.5	0.0	-14.5
B07	1.0	5.1	6.1	-5.4	-0.7	-6.1
B08	16.3	56.8	73.1	-52.9	-20.0	-73.0
B09	12.3	34.1	46.4	-35.3	-11.0	-46.3
B10	10.1	35.4	45.5	-36.6	-8.9	-45.5
B11	5.5	31.2	36.7	-30.6	-6.1	-36.7
B12	-13.8	25.6	11.8	-13.2	1.5	-11.7
B13	-10.9	90.8	80.0	-51.0	-28.9	-79.8
B14	1.5	9.0	10.5	-9.5	-1.0	-10.5
B15	0.4	27.0	27.4	-23.7	-3.7	-27.4
B16	-8.9	18.3	9.4	-10.7	1.3	-9.4
B17	3.4	14.6	18.0	-15.2	-2.8	-18.0
B18	5.2	18.1	23.3	-19.4	-3.9	-23.3
B19	1.9	10.8	12.7	-10.9	-1.8	-12.7
B20	10.3	34.7	45.0	-34.2	-10.9	-45.1
B21	-7.1	11.1	4.0	-5.7	1.7	-3.9
B22	9.2	25.6	34.7	-27.6	-7.2	-34.8
B23	-2.9	14.7	11.9	-11.5	-0.4	-11.9
B24	-3.2	12.2	9.0	-9.0	0.0	-9.0
B25	5.2	17.6	22.8	-19.1	-3.8	-22.9
B26	-0.1	6.8	6.7	-6.3	-0.4	-6.7
B27	NA	NA	NA	NA	NA	NA
B28	0.1	19.7	19.8	-17.4	-2.4	-19.8
B29	0.5	4.7	5.3	-4.9	-0.4	-5.3
B30	12.0	31.7	43.7	-32.7	-11.1	-43.9
B31	0.1	3.9	4.0	-3.8	-0.2	-4.0
B32	2.6	12.6	15.2	-12.9	-2.4	-15.3
B33	-4.1	22.0	18.0	-15.8	-2.2	-18.0
B34	0.6	3.9	4.6	-4.1	-0.5	-4.6
B35	NA	NA	NA	NA	NA	NA
B36	-8.6	35.6	26.9	-22.7	-4.2	-26.9
B37	-3.4	13.7	10.3	-10.4	0.1	-10.3
B38	0.0	6.4	6.4	-6.0	-0.4	-6.4
B39	0.6	6.7	7.3	-6.8	-0.6	-7.4
B40	0.1	2.1	2.2	-2.1	-0.1	-2.2
B41	-6.9	5.9	-1.0	-2.0	3.0	1.1
B42	-7.2	8.4	1.2	-3.1	1.7	-1.4
B43	-10.5	20.2	9.7	-13.0	3.2	-9.8
B44	-2.3	4.4	2.1	-2.8	0.5	-2.2
B45	-0.8	16.4	15.6	-13.9	-1.8	-15.7
B46	0.4	2.6	3.0	-2.8	-0.2	-3.0
B47	0.7	3.5	4.2	-3.9	-0.4	-4.2
B48	0.1	1.5	1.7	-1.6	-0.1	-1.7
B49	NA	NA	NA	NA	NA	NA

Table S5. Mean value for the terms in R^2P , R^2 and R^3 (Eq.4), subdivided according to the contributions of the terms that depend on p_e (that is, p_e and p_e^2), p_a (that is, p_a and p_a^2) or both ($p_e p_a$). Terms in R^2P depending on p_e (1st column), terms in R^2P depending on p_a (2nd column), terms in R^2P depending on $p_e p_a$ (3rd column), terms in R^2P depending on p_a or $p_e p_a$ (4th column). Terms in R^2 and R^3 depending on p_e (5th column), terms in R^2 and R^3 depending on p_a (6th column), terms in R^2 and R^3 depending on $p_e p_a$ (7th column), terms in R^2 and R^3 depending on p_a or $p_e p_a$ (8th column). The term in RP^2 does not appear on this table because it is independent from any combination of p_e and p_a .

Basin	$-R^2P(\beta_8 - \beta_5 p_e + \beta_6 p_e^2)$	$-R^2P(-\beta_7 p_a + \beta_9 p_a^2)$	$-R^2P(\beta_8 p_e p_a)$	$-R^2P(-\beta_7 p_a + \beta_8 p_e p_a + \beta_9 p_a^2)$	$+R^2(\beta_1 - \beta_2 p_e + \beta_3 p_e^2) + R^3(\beta_{10} - \beta_{11} p_e + \beta_{12} p_e^2)$	$R^3(-\beta_{13} p_a + \beta_{14} p_a^2)$	$R^3(\beta_{14} p_e p_a)$	$R^3(-\beta_{13} p_a + \beta_{14} p_e p_a + \beta_{15} p_a^2)$
B01	3.5	206.4	-206.3	0.0	-36.4	-167.3	169.3	2.0
B02	-6.0	39.3	-31.4	7.9	-13.7	-27.0	21.7	-5.3
B03	NA	NA	NA	NA	NA	NA	NA	NA
B04	0.8	25.6	-26.8	-1.2	-14.2	-15.9	16.7	0.8
B05	-68.4	119.7	-83.7	36.0	8.3	-80.5	56.6	-23.9
B06	-10.7	18.3	-13.5	4.8	-11.8	-10.5	7.8	-2.7
B07	0.8	2.5	-2.4	0.2	-6.0	-1.2	1.1	-0.1
B08	-7.3	142.4	-118.7	23.7	-56.9	-99.0	83.0	-16.1
B09	1.8	85.4	-74.8	10.6	-39.9	-53.7	47.3	-6.4
B10	8.0	40.9	-38.8	2.1	-44.3	-24.0	22.8	-1.2
B11	4.9	31.2	-30.7	0.5	-36.4	-17.5	17.2	-0.2
B12	-11.9	18.2	-20.0	-1.9	-12.7	-9.7	10.7	1.0
B13	-124.9	685.1	-571.1	114.0	4.6	-542.6	458.1	-84.5
B14	1.1	2.9	-2.5	0.4	-10.4	-1.3	1.1	-0.2
B15	1.0	26.2	-26.7	-0.6	-27.8	-14.7	15.1	0.4
B16	-6.1	24.4	-27.3	-2.8	-10.9	-12.7	14.3	1.5
B17	3.0	4.7	-4.3	0.4	-17.8	-2.6	2.4	-0.2
B18	3.7	19.4	-18.0	1.4	-22.6	-10.5	9.8	-0.7
B19	1.7	11.1	-10.9	0.2	-12.6	-5.9	5.8	-0.1
B20	-3.6	90.1	-76.2	13.9	-36.5	-57.5	48.9	-8.6
B21	-8.6	4.7	-3.2	1.5	-3.2	-2.4	1.7	-0.8
B22	3.6	56.7	-51.1	5.6	-31.7	-33.2	30.1	-3.1
B23	-1.4	19.4	-20.8	-1.4	-12.7	-9.7	10.5	0.8
B24	-6.3	11.6	-8.5	3.1	-7.5	-5.8	4.3	-1.5
B25	3.5	28.5	-26.8	1.7	-22.0	-15.2	14.4	-0.8
B26	-0.9	4.0	-3.1	0.9	-6.3	-1.8	1.4	-0.4
B27	NA	NA	NA	NA	NA	NA	NA	NA
B28	-4.8	22.8	-17.9	4.9	-17.1	-13.0	10.2	-2.8
B29	0.5	1.7	-1.6	0.1	-5.3	-0.7	0.7	0.0
B30	0.2	106.8	-95.0	11.8	-36.6	-69.6	62.3	-7.3
B31	-0.1	0.8	-0.6	0.2	-3.9	-0.3	0.3	-0.1
B32	-0.5	19.0	-15.9	3.1	-13.6	-10.2	8.6	-1.6
B33	-16.2	48.5	-36.4	12.2	-10.9	-29.0	21.9	-7.2
B34	-0.7	9.0	-7.7	1.3	-4.0	-4.0	3.5	-0.6
B35	NA	NA	NA	NA	NA	NA	NA	NA
B36	-31.8	90.8	-67.6	23.2	-11.6	-61.1	45.7	-15.4
B37	-1.1	23.6	-25.9	-2.3	-11.5	-11.5	12.7	1.2
B38	0.0	2.8	-2.8	0.0	-6.5	-1.2	1.2	0.0
B39	0.1	3.0	-2.5	0.5	-7.1	-1.3	1.1	-0.2
B40	0.1	0.7	-0.7	0.0	-2.2	-0.2	0.2	0.0
B41	-9.0	6.0	-3.9	2.1	3.0	-5.3	3.4	-1.9
B42	-0.2	18.8	-25.8	-7.0	-3.3	-4.6	6.4	1.8
B43	-18.2	138.8	-131.0	7.7	-6.7	-86.2	83.1	-3.1
B44	0.2	9.7	-12.2	-2.5	-2.9	-2.5	3.2	0.7
B45	-9.2	91.5	-83.1	8.4	-11.4	-55.2	50.9	-4.3
B46	0.1	2.2	-2.0	0.3	-2.9	-0.8	0.7	-0.1
B47	0.1	5.7	-5.1	0.6	-4.0	-2.3	2.0	-0.2
B48	0.1	1.3	-1.3	0.0	-1.7	-0.4	0.4	0.0
B49	NA	NA	NA	NA	NA	NA	NA	NA

Table S6. Mean value for each term of Eq.3: terms in R^2P (1st column), terms in RP^2 (2nd column), sum of terms in R^2P and RP^2 (3rd column) and terms in R^2 (4th column).

Basin	$-(\theta_4-\theta_5p_e)R^2P$	$(\theta_1p_e-\theta_2p_e^2)RP^2$	$(\theta_1p_e-\theta_2p_e^2)RP^2$ $-(\theta_4-\theta_5p_e)R^2P$	$-\theta_3R^2$
B01	-6.5	36.9	30.4	-30.4
B02	-5.5	22.3	16.8	-16.8
B03	-6.5	15.1	8.5	-8.5
B04	1.0	4.5	5.4	-5.4
B05	-20.6	59.0	38.5	-38.4
B06	-7.6	29.3	21.6	-21.6
B07	0.1	3.6	3.7	-3.7
B08	-14.7	67.4	52.7	-52.6
B09	-5.5	37.5	32.0	-32.0
B10	1.2	24.7	25.9	-25.9
B11	2.0	15.5	17.5	-17.6
B12	0.1	0.4	0.5	-0.6
B13	-35.3	100.0	64.6	-64.6
B14	-0.5	9.4	8.9	-8.9
B15	1.7	8.0	9.6	-9.7
B16	0.2	0.7	0.9	-1.0
B17	0.2	11.2	11.4	-11.4
B18	0.0	14.5	14.5	-14.5
B19	0.5	5.5	6.0	-6.0
B20	-7.7	40.7	33.0	-33.1
B21	-4.6	17.1	12.5	-12.4
B22	-2.5	26.2	23.7	-23.8
B23	0.4	2.1	2.5	-2.6
B24	-3.9	17.2	13.3	-13.4
B25	-0.1	14.2	14.1	-14.2
B26	-1.4	9.5	8.1	-8.1
B27	0.0	0.0	0.0	0.0
B28	-5.5	26.5	21.0	-21.1
B29	0.1	2.2	2.3	-2.3
B30	-5.9	35.9	30.0	-30.1
B31	-0.6	5.5	4.9	-4.9
B32	-2.2	15.4	13.2	-13.3
B33	-7.3	28.0	20.7	-20.7
B34	-0.7	5.1	4.4	-4.4
B35	0.0	-0.1	0.0	-0.1
B36	-14.1	45.9	31.9	-31.8
B37	0.4	2.0	2.3	-2.4
B38	0.2	1.4	1.6	-1.6
B39	-0.9	8.6	7.7	-7.7
B40	0.0	0.7	0.7	-0.7
B41	-1.1	5.0	3.9	-3.9
B42	0.8	3.6	4.4	-4.7
B43	-7.5	26.8	19.3	-19.4
B44	0.2	2.3	2.5	-2.6
B45	-4.9	20.8	15.9	-16.0
B46	-0.2	3.0	2.9	-2.9
B47	-0.4	4.3	3.9	-3.9
B48	0.0	1.1	1.1	-1.1
B49	-1.4	5.3	3.8	-3.8

FIGURES

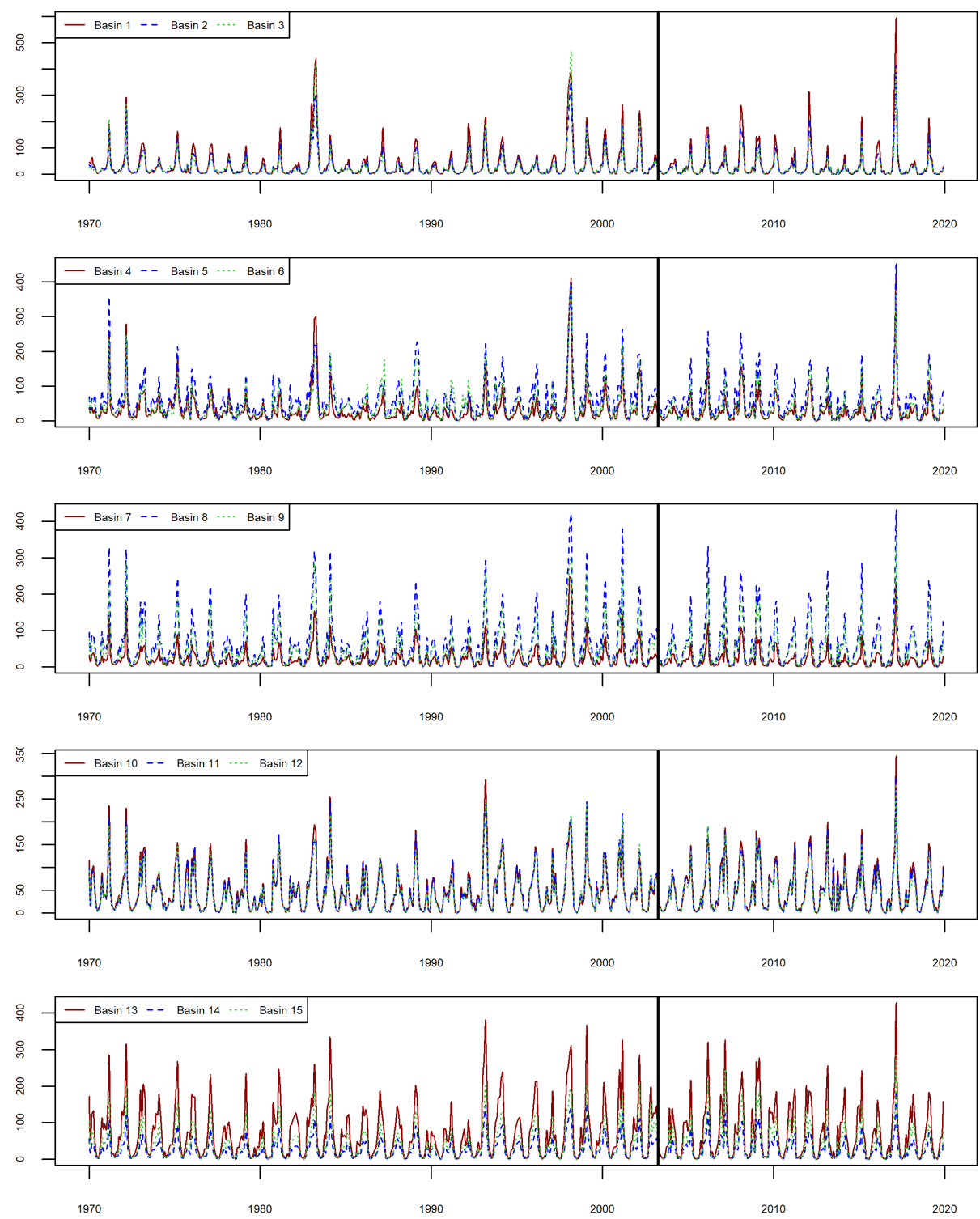


Figure S1. Monthly rainfall (in mm) observed at Basins B01 to B15.

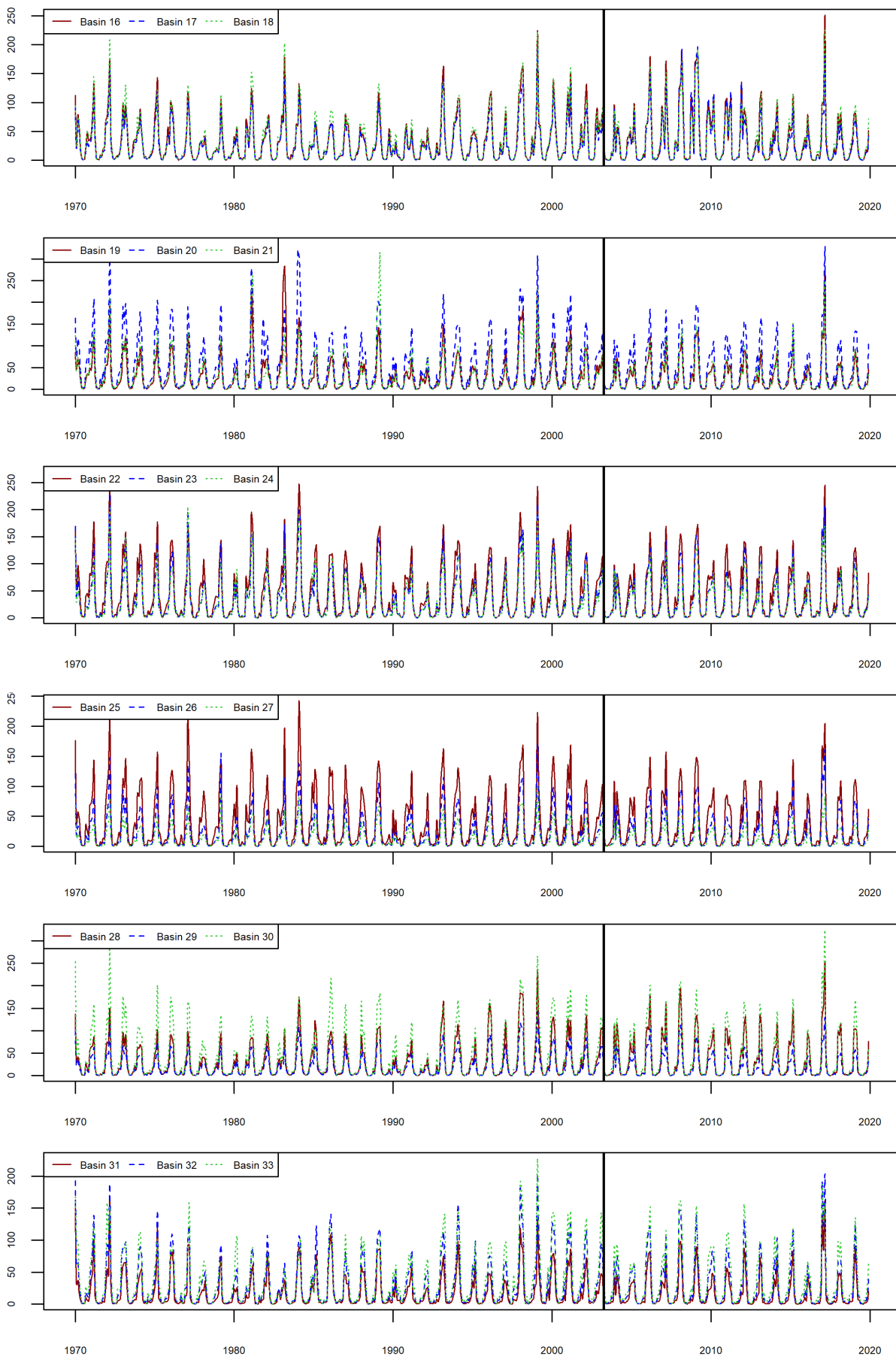


Figure S2. Monthly rainfall (in mm) observed at Basins B16 to B33.

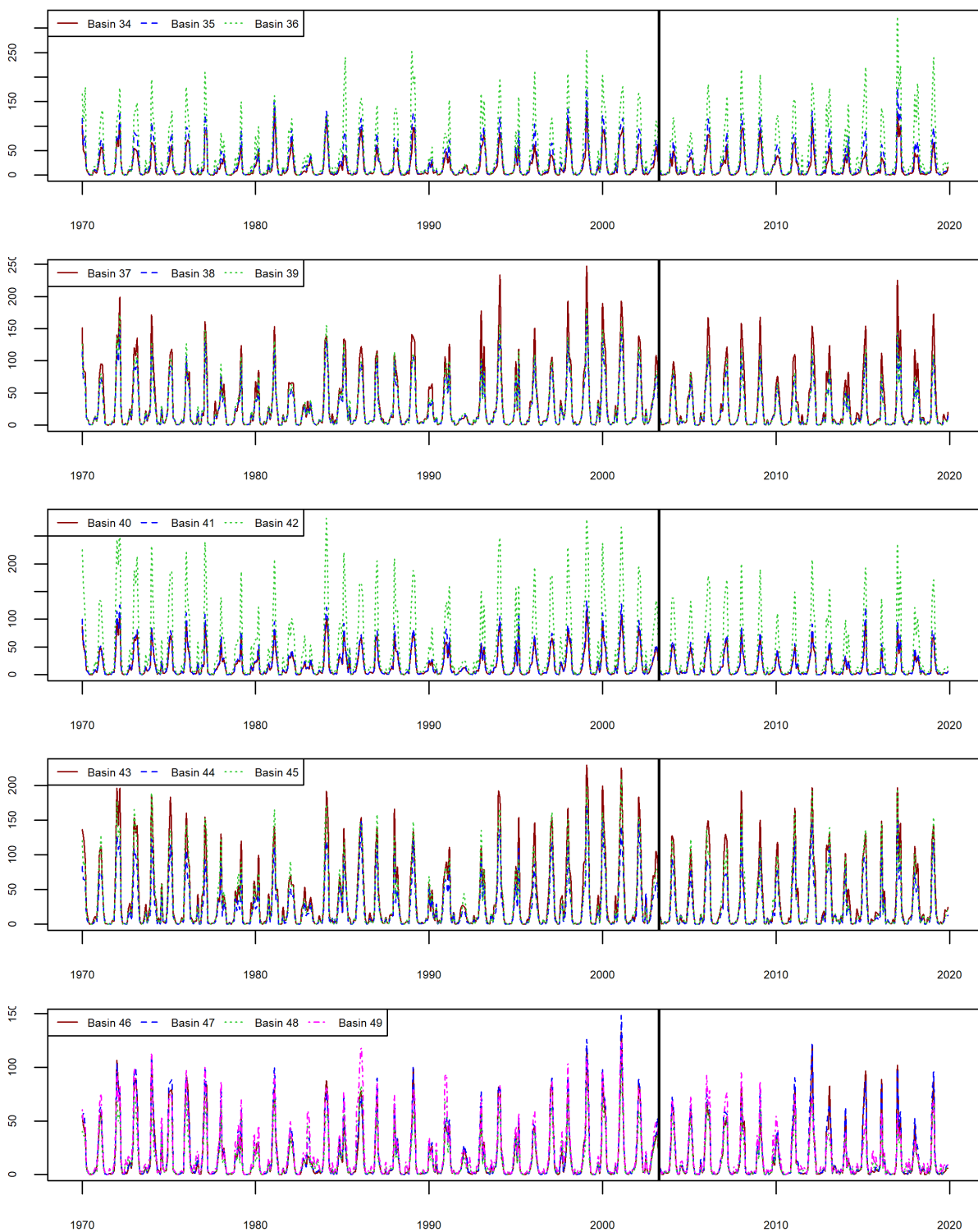


Figure S3. Monthly rainfall (in mm) observed at Basins B34 to B49.

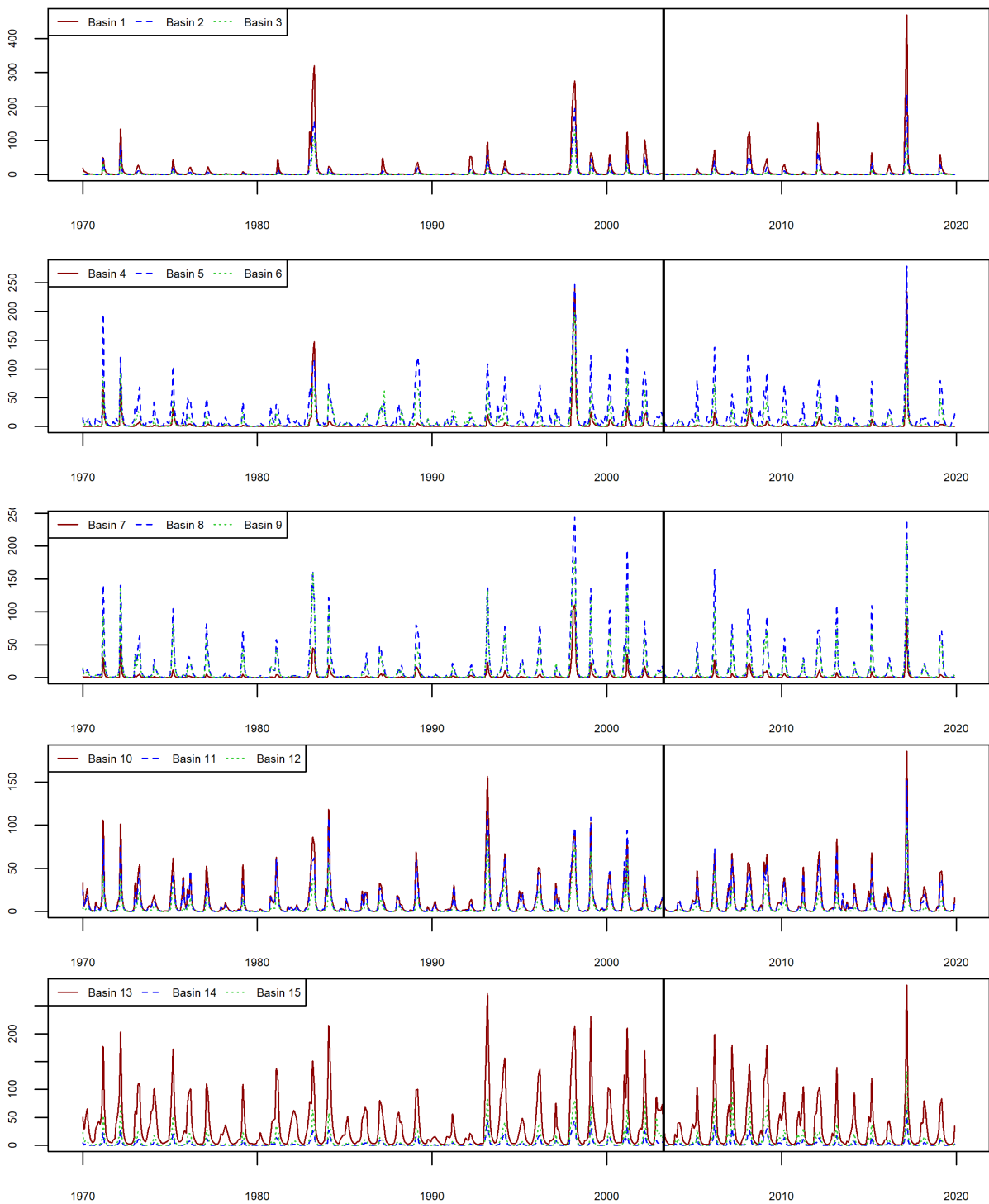


Figure S4. Monthly runoff (in mm) observed at Basins B01 to B15.

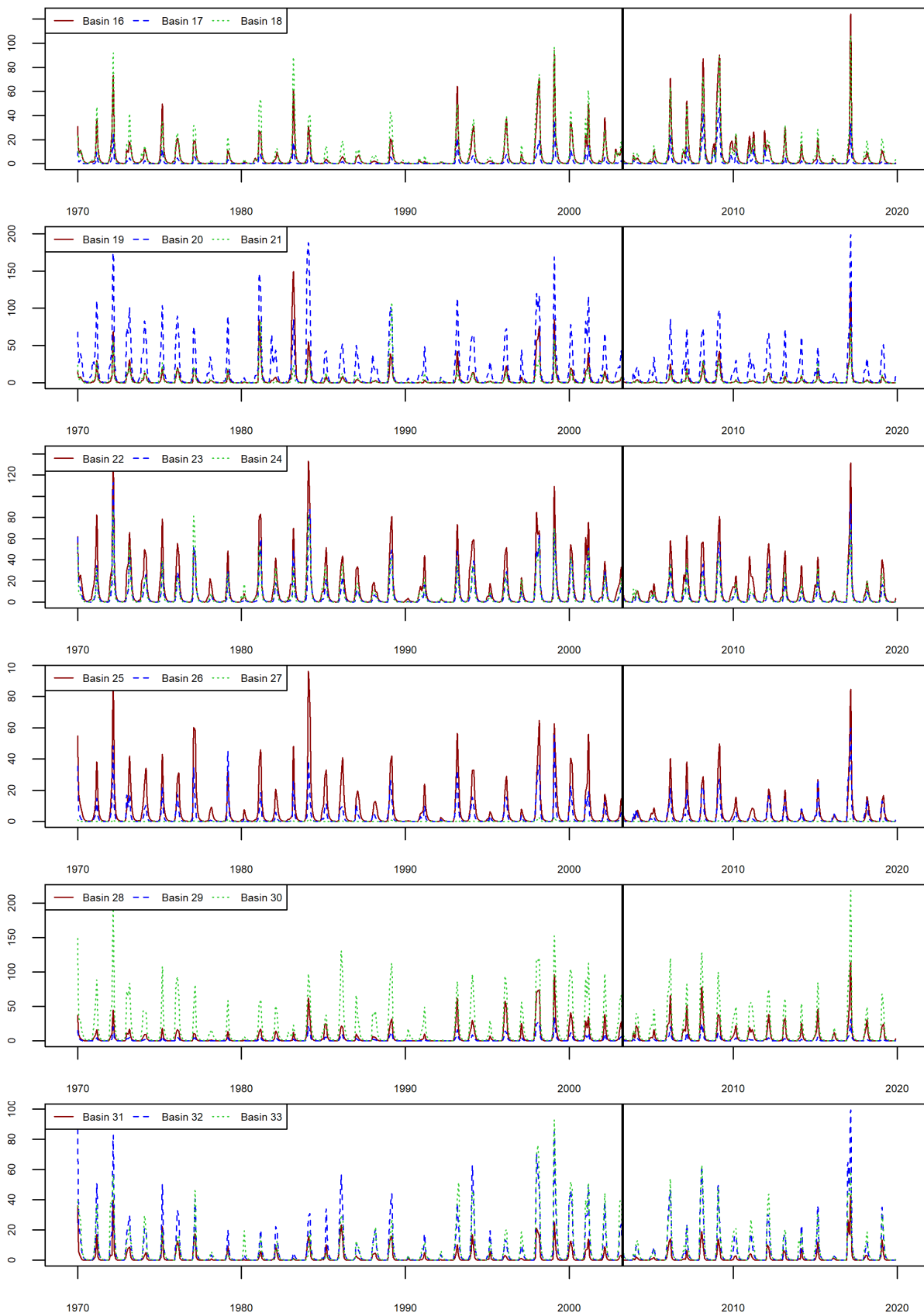


Figure S5. Monthly runoff (in mm) observed at Basins B16 to B33.

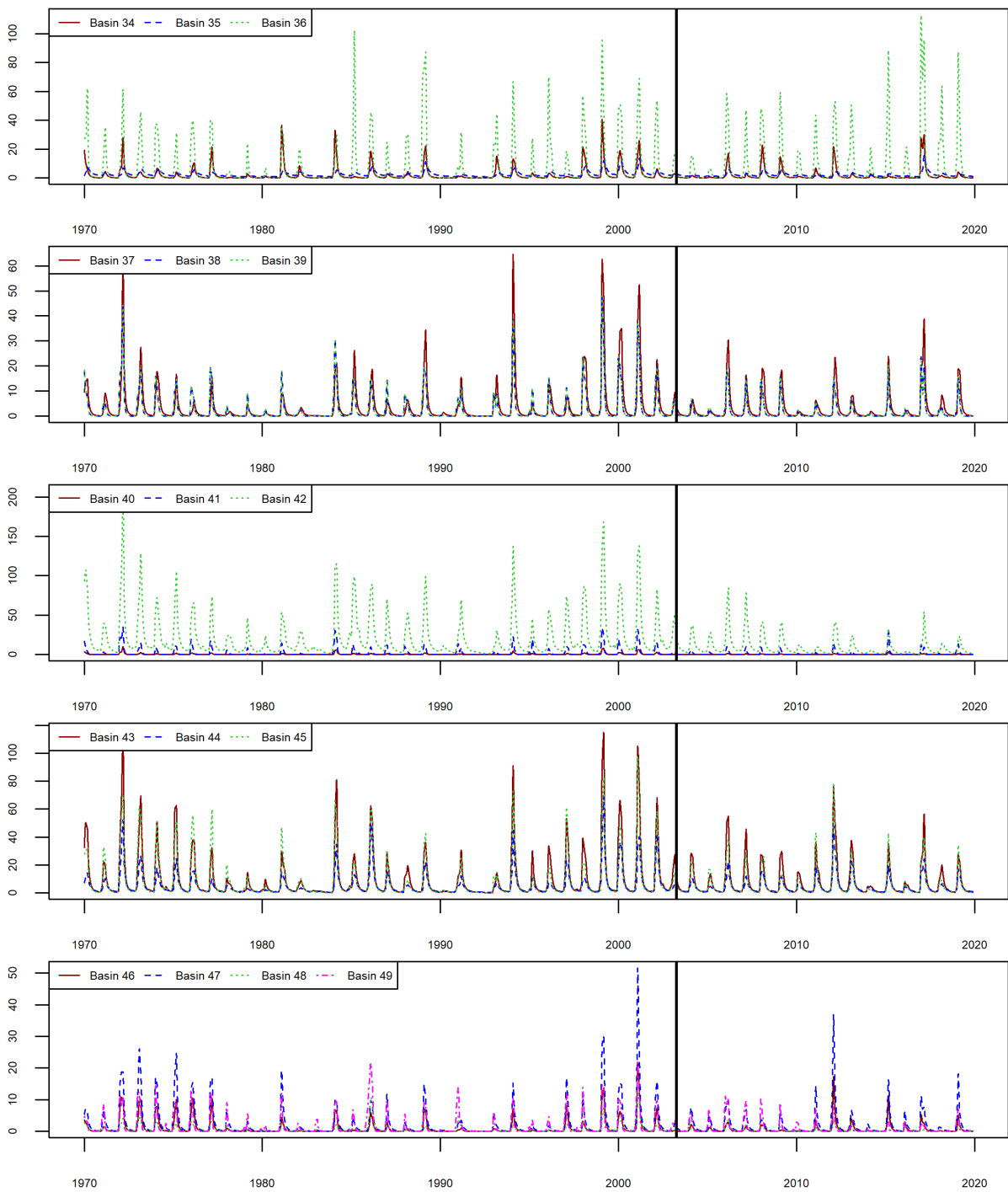


Figure S6. Monthly runoff (in mm) observed at Basins B34 to B49.

Due Date of Revised Submission: 12-Apr-2026

Editor's comments:

Reviewer(s)' Comments to Author:

Reviewer: 1

Comments to the Author

In the manuscript “new class of monthly rainfall-runoff model for ungauged basins – Application to the Peruvian coastal basins”, the authors present an application of the GPoM package. The approach is data-driven but relies on physically meaningful parameters to reconstruct rainfall-runoff dynamics across 49 basins in Peru. The topic is relevant for Hydrological Processes readership and the methods are sound. However, some clarifications are needed. I recommend minor revision before the manuscript can be considered for publication.

This is a good summary of the study.

My minor comments are as follows:

Lines 56-57: rainfall--runoff -> rainfall-runoff.

It has been changed accordingly, thank you.

Section 2.1: Please consolidate the description of El Niño/La Niña in Section 2.1 and avoid spreading it across later sections.

Changed accordingly, thank you.

Section 2.4: It is difficult to understand why higher temporal resolution data (at least daily) were not used for rainfall-runoff modelling. In my view, monthly data are too coarse to adequately capture the dynamics of the rainfall-runoff transformation, especially for extreme events. Moreover, the applied filtering/smoothing does not add physically realistic information; rather, it mainly increases the amount of data available to the data-driven model. By reconstructing the time series using smoothing filters, you may be suppressing important variability and missing the dynamics of extremes. Please (i) justify the choice of monthly data, and (ii) explicitly discuss the associated limitations later in the manuscript.

This is an interesting remark. Higher temporal resolution data would very likely improve the results in terms of predictions, in particular the estimates of extreme events. However, it would probably also complexify the models which would make them more difficult to interpret. In addition, this study covers a large temporal and spatial extent and the availability of higher resolution data is limited. Therefore, we would prefer to keep the analysis of higher resolution data for a later study. We added some comments in the paper to explain it (see Section 2.4 “While higher-resolution [...] interpretability” and just before starting the Discussion “A finer temporal resolution [...] for future work”).

Section 3.1: Please define $\text{CE}=\text{B5}$ in Equation (1).

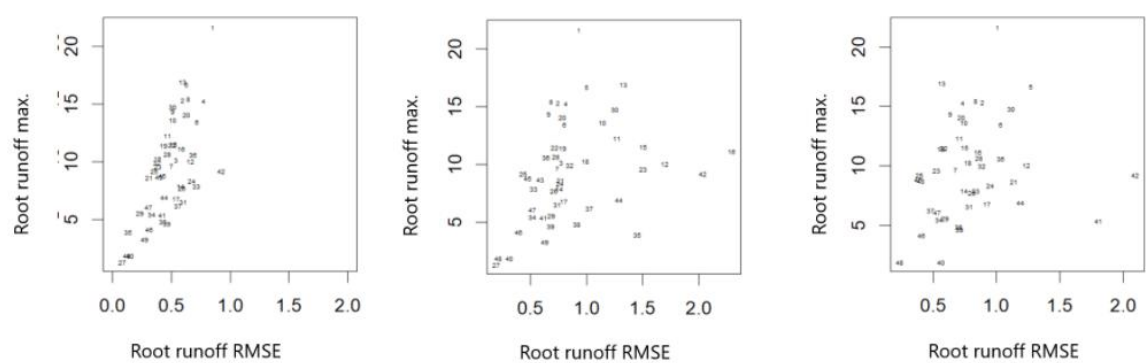
Indeed, some terms were not described. Now corrected. Thank you for that.

Section 3.2: Please explain more clearly why concatenated time series are needed for calibration across multiple basins. Operationally, this is a key step to move from single-catchment calibration to a multi-catchment assessment, and it is worth describing more explicitly.

Indeed, it is a key step, here. The concatenation is now explained more carefully in Section 3.2, see in particular “To do so, the time series of each variable were concatenated sequentially to create a single extended precipitation time series. Basically, concatenation was performed by assembling the time series of the modelling basins one after the other, as illustrated in Figure 2 (here considering four modelling basins). This way, we get four time series: the first one for $R(t)$, the second one for $P(t)$, the third one for $p_a(t)$ and the fourth one for $p_e(t)$. This way, each variable includes the concatenated information of the four basins used for the modelling (B01, B02, B03 and B04, respectively in purple, pink, cyan and blue colours). For each basin, the area and the elongation parameters are constant in time, but they vary spatially from one basin to another. A gap was inserted between each consecutive series in order to avoid spurious values in the derivatives.”. Thank you.

Section 3.4: The final metric mentioned in Lines 292-293 still tends to overweight high flows. It would be useful to include alternative metrics based on log or Box-Cox transformations to assess whether the models also perform well during dry years (low-flow conditions). I understand that you may have many near-zero or zero discharge values, but it is still worth exploring and discussing this point.

This is an interesting suggestion. We have calculated the following metrics in order to better assess the errors, especially in low flow conditions: square root, boxcox with $\lambda = 0$ (equivalent to log transformation), $\lambda = 0.2$ and $\lambda = 0.5$. Using the square root of RMSE is appropriate, as the modelling process was conducted on square-root-transformed data. We have added some comments on the root runoff RMSE to the results section “The RMSE calculated from the root runoff [...] for these basins”.



The log and Box–Cox results were very similar to the RMSE; therefore, we did not include them in the manuscript to avoid overloading it.

BASIN	INDIVIDUAL MODEL					MODEL GI					MODEL GII				
	Original	Root	Log	Lambda = 0.2	Lambda = 0.5	Original	Root	Log	Lambda = 0.2	Lambda = 0.5	Original	Root	Log	Lambda = 0.2	Lambda = 0.5
1	10.34	0.85	0.46	0.71	1.64	11.47	0.93	0.49	0.76	1.80	12.36	1.01	0.52	0.82	1.95
2	4.52	0.60	0.36	0.51	1.01	5.73	0.75	0.43	0.61	1.22	6.63	0.89	0.53	0.74	1.44
3	4.44	0.54	0.35	0.49	0.97	2.54	0.77	0.56	0.67	0.97	NA	NA	NA	NA	NA
4	8.97	0.77	0.44	0.58	1.22	11.03	0.82	0.42	0.64	1.52	4.55	0.73	0.51	0.63	1.01
5	5.37	0.63	0.42	0.60	1.18	13.03	1.00	0.47	0.78	1.97	12.19	1.27	0.63	0.94	2.05
6	4.55	0.72	0.51	0.70	1.25	6.18	0.80	0.45	0.66	1.35	8.07	1.04	0.59	0.85	1.73
7	3.02	0.50	0.35	0.47	0.82	5.25	0.74	0.47	0.65	1.23	4.25	0.68	0.44	0.58	1.03
8	6.46	0.64	0.35	0.51	1.10	6.48	0.68	0.36	0.54	1.15	7.96	0.84	0.42	0.64	1.40
9	4.86	0.51	0.31	0.45	0.94	7.64	0.66	0.31	0.51	1.27	6.37	0.64	0.32	0.51	1.17
10	4.24	0.51	0.32	0.48	0.96	11.73	1.14	0.61	0.98	2.25	7.23	0.74	0.40	0.63	1.40
11	3.69	0.47	0.31	0.45	0.88	12.04	1.27	0.71	1.13	2.49	6.30	0.71	0.39	0.60	1.28
12	4.07	0.66	0.49	0.65	1.13	9.49	1.69	1.27	1.70	2.89	6.08	1.24	0.79	1.05	1.74
13	8.10	0.60	0.26	0.42	1.10	19.59	1.33	0.58	0.96	2.55	7.91	0.57	0.26	0.42	1.08
14	2.10	0.58	0.47	0.59	0.87	4.29	0.76	0.52	0.71	1.26	4.13	0.75	0.52	0.71	1.23
15	4.10	0.52	0.35	0.50	0.95	13.55	1.50	0.90	1.37	2.90	6.37	0.75	0.44	0.64	1.32
16	4.90	0.58	0.39	0.56	1.10	16.37	2.29	1.53	2.17	4.08	8.69	0.85	0.44	0.69	1.58
17	2.45	0.54	0.42	0.55	0.88	3.70	0.80	0.57	0.72	1.16	4.96	0.92	0.65	0.84	1.40
18	2.95	0.38	0.25	0.36	0.70	8.47	0.99	0.61	0.92	1.89	6.24	0.78	0.49	0.72	1.45
19	3.25	0.43	0.29	0.40	0.73	6.94	0.79	0.48	0.71	1.45	3.83	0.57	0.36	0.50	0.90
20	4.62	0.63	0.43	0.61	1.15	8.63	0.78	0.41	0.66	1.54	6.83	0.72	0.41	0.64	1.40
21	1.43	0.31	0.23	0.29	0.47	2.99	0.77	0.54	0.69	1.08	4.31	1.14	0.82	1.02	1.53
22	4.30	0.51	0.32	0.47	0.97	7.06	0.71	0.39	0.62	1.39	5.19	0.59	0.35	0.54	1.13
23	2.83	0.38	0.25	0.36	0.70	10.33	1.50	1.03	1.48	2.78	4.16	0.53	0.29	0.43	0.89
24	3.16	0.67	0.52	0.68	1.11	5.04	0.76	0.49	0.70	1.35	5.77	0.95	0.61	0.86	1.60

25	2.49	0.36	0.21	0.30	0.59	3.47	0.44	0.24	0.37	0.77	2.20	0.39	0.23	0.33	0.61
26	2.38	0.59	0.47	0.60	0.91	3.74	0.71	0.50	0.67	1.17	4.29	0.81	0.57	0.77	1.34
27	0.08	0.08	0.04	0.05	0.06	0.15	0.20	0.11	0.11	0.13	NA	NA	NA	NA	NA
28	2.77	0.46	0.33	0.46	0.81	5.13	0.72	0.46	0.66	1.27	5.95	0.86	0.55	0.78	1.50
29	0.92	0.23	0.19	0.23	0.35	3.09	0.69	0.51	0.68	1.12	2.27	0.60	0.43	0.54	0.84
30	5.17	0.51	0.27	0.42	0.94	14.54	1.26	0.62	1.01	2.46	12.20	1.12	0.58	0.93	2.18
31	2.14	0.60	0.47	0.59	0.88	2.87	0.74	0.53	0.69	1.08	3.19	0.79	0.57	0.74	1.19
32	3.05	0.38	0.23	0.34	0.69	7.15	0.85	0.48	0.74	1.59	7.09	0.88	0.52	0.79	1.64
33	4.18	0.71	0.51	0.70	1.24	3.84	0.53	0.29	0.43	0.89	4.73	0.84	0.47	0.66	1.22
34	1.79	0.33	0.24	0.33	0.57	2.91	0.52	0.37	0.52	0.93	2.97	0.55	0.38	0.54	0.97
35	0.55	0.13	0.16	0.19	0.26	2.62	1.45	0.80	0.92	1.17	NA	NA	NA	NA	NA
36	6.10	0.68	0.42	0.57	1.14	5.77	0.64	0.35	0.51	1.11	8.27	1.04	0.59	0.84	1.68
37	2.74	0.56	0.43	0.57	0.93	5.50	1.02	0.76	1.06	1.87	2.17	0.48	0.33	0.44	0.72
38	1.89	0.43	0.31	0.42	0.69	3.79	0.91	0.73	0.96	1.55	2.65	0.71	0.50	0.64	1.02
39	1.84	0.46	0.32	0.42	0.68	2.51	0.68	0.48	0.62	0.98	2.65	0.71	0.51	0.65	1.03
40	0.16	0.15	0.09	0.10	0.11	0.30	0.31	0.17	0.19	0.22	0.80	0.56	0.40	0.44	0.53
41	1.57	0.42	0.32	0.41	0.64	2.14	0.61	0.44	0.56	0.86	5.23	1.81	1.41	1.74	2.46
42	7.83	0.92	0.52	0.80	1.68	13.64	2.03	1.31	1.86	3.50	14.11	2.09	1.35	1.93	3.65
43	2.98	0.40	0.25	0.36	0.70	4.29	0.59	0.41	0.56	1.03	3.12	0.41	0.24	0.36	0.73
44	2.78	0.44	0.31	0.44	0.79	6.13	1.29	0.92	1.23	2.05	5.94	1.19	0.84	1.13	1.92
45	3.17	0.42	0.26	0.39	0.78	3.39	0.48	0.36	0.49	0.88	2.74	0.38	0.26	0.38	0.72
46	0.98	0.31	0.24	0.30	0.44	1.06	0.40	0.29	0.35	0.49	1.11	0.41	0.31	0.37	0.52
47	1.36	0.31	0.25	0.32	0.50	2.54	0.51	0.41	0.55	0.91	2.58	0.53	0.43	0.57	0.94
48	0.17	0.12	0.09	0.10	0.12	0.34	0.21	0.17	0.19	0.24	0.31	0.24	0.15	0.17	0.20
49	0.60	0.27	0.21	0.25	0.33	1.38	0.63	0.45	0.53	0.72	NA	NA	NA	NA	NA

Sections 4 and 5: The titles of the main sections are confusing. Presenting Results before Simulations is counterintuitive. In addition, the Results subsections are very technical and difficult to follow. Please consider being more concise and focusing on the key findings.

We agree with the reviewer. The order of these sections has been exchanged. Text has been simplified and now focuses on the main results. The simulation experiments are now explained in Appendix and discussed in the Discussion Section which makes it more fluent for the reader. Thank you for this helpful comment.

Section 6: A strength of this section is that it attempts to interpret hydrological processes a posteriori, which can also help reject unsuitable model structures. This aspect should be highlighted more clearly, especially for a journal like Hydrological Processes.

This is an interesting suggestion, thank you. A small paragraph was added at the beginning of the discussion (now section 5) to underline this idea, and at the end when explaining Figure 9 (see, in particular, the last sentence of this section “Thus, the analysis of these patterns provided a basis for interpreting the underlying hydrological processes and rejecting unsuitable model structures.”).

Conclusion: The claim of a “new class” of rainfall-runoff models is not sufficiently developed in the manuscript. It is mentioned only in the abstract and conclusion, and the difference from more classic data-driven approaches may not be clear to readers. Please clarify - at the end of the Introduction and reiterate in the Conclusion - why this should be considered a new class and how it differs from other data-driven approaches (in line with what you outline in the Novelty statement).

You are perfectly right, the claim of new class of models was not explained. Thank you for this suggestion. Usually, the global modelling technique is applied locally (at a given geographical location) to obtain models that reproduces an observed dynamic at this place. The approach is insensitive to the initial conditions and independent from the forcing, but it is ad hoc, in the sense that the model is specific to the studied location. In the present study, the technique is generalized: it is applied to reproduce the global dynamics not just for a chosen location with a given parameterization (that is: a particular case of study at one location) but including the influence of parameters which can vary in space, and therefore it can apply to a continuous ensemble of parameters (p_e, p_a) which makes it a new class of global models, and more specifically here a new class of rainfall-runoff models. A paragraph was added in Section 3.1 (after Eq. 2) to explain in what it is a new class of models (*“This new formulation aims to generalize the usual global modelling technique. In most of the previous studies, this technique was applied locally, or, at best, assuming that the spatial variability could be resumed as a single dynamic, either by aggregating the observations as a homogeneous signal, or by considering the time series in association, that is, as an ensemble of dynamics possibly independent (in phase, in initial conditions) but all identical (same equations and parameterizations). This problem was investigated both theoretically by considering ensembles of Rössler systems (Mangiarotti et al. 2016) or practically for the cycles of cereal crops observed by satellite in Morocco (Mangiarotti and Le Jean 2023). The formulation presented here in Eq. (2) is still independent from the initial conditions and the forcing, but more general by reconstructing not only the original set of equations but also parameter dependence. This is what makes this approach a new class of global models because it aims at reproducing not only the global dynamics as a particular case, but the global dynamics as general case taking also in account the variations of its parameters. Of course, only the role of the parameters provided as an input can be expected to be involved in the reconstructed set of equations.”*). The global modelling technique was rarely applied to hydrology. It was applied to the discharge of karst springs and to soil ecohydrology, without forcing; and to the aero-hydrology of caves with forcing. But it was not applied in the general case with the explicit parameter dependence taken in account. This is now explained more carefully in Section 3.1 and this result was recalled and summarized in the Conclusion. Thank you for this useful and important suggestion.

Conclusion (Line 597): I suggest adding that the remarkably consistent dynamical pattern is ‘based on catchment morphology.’

This is an important point, indeed. Done

General comment: As an additional strength of the framework, please state somewhere that the only base layer needed for the model is a DTM. This is an important aspect of the model’s parsimony. In my view, parsimony is not only about the number of parameters, but also about computational cost and data requirements.

You are perfectly right, a DTM will be sufficient to parameterize the model (to estimate the watershed surface and elongation), and even a rough DTM may be sufficient, a sentence was added

to clarify this point at the end of the Introduction (*"It should be noted that model parameterization requires only two parameters that can be directly derived from a Digital Terrain Model (DTM) of the basin, and the models are also computationally efficient."*). And indeed, such an approach is also quite frugal in terms of computation resources. Thank you.

In conclusion, I recommend that the authors undertake a minor revision, and I would welcome reviewing the revised manuscript to assess how these issues have been addressed.

Thank you very much for your very constructive and useful review. We hope we were able to answer all your concerns, satisfyingly.

Reviewer: 2

Comments to the Author

The global modelling technique developed further and applied in this manuscript seems to be an approach worthy of informing hydrologists as to its existence and application potential. The manuscript seems to represent a first attempt to achieve this.

This is indeed the case, we agree with the referee.

However I found the large amount of work undertaken here to be unclear in necessary detail as to what was actually done in the application.

Many points have been clarified. The reviews have been very useful to us and we would like to thank the referees sincerely for their careful reading and constructive comments.

What specifically is the potential use of these models? It seems to be for computing monthly runoff in ungauged basins and for climate forcing scenarios effect on runoff. How good is the modelling approach when forced by historical data with different climate signatures. How good is the model for ungauged basins (ie are some basins excluded from the analysis and treated as ungauged and then the method's efficacy tested on them?)

Indeed, in this paper, it is shown that the models (1) can be obtained from a given historical period (1970-2003) and can then be applied to another historical period (2003-2019 is used only for validation, in all the cases); and (2) can be obtained from some basins (e.g. B01 to B04 on 1970-2003 for G_I) and can then be applied to other basins on other periods (B05 to B49 used only for validation on 2003-2019). The same was done with G_{II} but considering 17 modelling basins (B01, B04, B07, B10, B13, B16, B19, B22, B25, B28, B31, B34, B37, B40, B43, B46, B49) and was then applied to the other 32 ones for validation (on 2003-2019). These points have been clarified (see Section 3.2) in the paper. The potential use of the models is now in the Introduction Section. Note that, for the generalized models (G_I and G_{II}), the fragment of time series were also separated (1970-2003 for modeling and

2003-2019 for validation), and that, in addition, all the basins were indeed not used for modelling (see Table 2 for details). Note that the RMSE error provided in Tables S2 and S3 are systematically computed on the validation period (2003-2019) and that gauged and ungauged basins are distinguished in these tables (gauged basins are in gray).

In general it is difficult to determine how testing was performed on independent periods and catchments for validation. Also, there seems to be many ad hoc decisions in the approach such as excluding catchments that do not perform well.

Actually, not at all. There were -- no ad hoc decision -- in the approach to exclude catchments that did not perform well. The same approach was applied systematically: always the same modelling period, always the same validation period in all cases (individual models, G_I model and G_{II} model). For individual analyses, all the basins were considered separately (no ungauged basins). For G_I we considered B01, B02, B03 and B04 for modelling and the other ones B05 to B49 for validation (systematically). And for G_{II} we considered one basin every three basins very systematically also (that is basin no $1+3n$ with $n = 0$ to 16 for modelling, and the other ones for validation). We have improved section 3.4 and added a section 3.5 in order to clarify these points, in addition to Table 2 (in which all the information is synthesized). On another hand, no catchment was excluded a priori; in the results, we just acknowledge that the models exhibit different levels of performance on different basins. All this is now explicitly mentioned in Section 3.2 (*"Note that the modelling and validation periods, and the modelling and validation basins are all synthesized in Table 2. There was no ad hoc decision to include or exclude any catchments: these choices were applied systematically and were not changed in the modelling-validation process."*). Thank you for this remark, it was indeed important to make it clear!

Section 3 and 3.3 in particular need much more explanation and clarity for those not familiar with the global modelling technique.

This section was not clear enough, we have brought additional details to clarify this point (in all the subsections of Sections 3).

Other more detailed examples that confused are:

Why square root transformation only for precipitation and runoff time series? How was this decided? Were other options tested?

We tested three approaches—log transformation, no transformation, and square root transformation—but only the square root transformation produced numerically stable models. A phrase has been added to the Preprocessing section to make it explicit (*"We tested three approaches—log transformation, no transformation, and square root transformation—but only the square root transformation produced numerically stable models."*).

"To achieve this, precipitation time series for all basins involved in the modelling were concatenated," HOW was this done?

It is indeed an important point worth being explained more carefully. Actually, what we mean by concatenated, is that the time series were just put one after the other, as illustrated in Figure 2 (here

considering 4 basins). This way, we get four time series: one for $R(t)$, one for $P(t)$, one for $p_a(t)$ and one $p_e(t)$ and each time series includes the concatenated information of the four basins used for modelling (B01, B02, B03 and B04, respectively in purple, pink, cyan and blue color).

This has been clarified in Section 3.2. (see in particular: *"To do so, the time series of each variable were concatenated sequentially to create a single extended precipitation time series. Basically, concatenation was performed by assembling the time series of the modelling basins one after the other, as illustrated in Figure 2 (here considering four modelling basins). This way, we get four time series: the first one for $R(t)$, the second one for $P(t)$, the third one for $p_a(t)$ and the fourth one for $p_e(t)$. This way, each variable includes the concatenated information of the four basins used for the modelling (B01, B02, B03 and B04, respectively in purple, pink, cyan and blue colours). For each basin, the area and the elongation parameters are constant in time, but they vary spatially from one basin to another. A gap was inserted between each consecutive series in order to avoid spurious values in the derivatives."*)

Show the factors in 2^{*30} . Ditto 2^{*10}

Actually, there was a mistake in the text because originally we wanted to contextualize the example with two variables instead of three, and we did not update all the numbers. We apology for that. We updated all the values and clarified the text in Section 3.3 (*"In the original GPoM algorithm, the model search for a two-dimensional system (one equation for the runoff and one for the precipitation) with maximum polynomial degree of three, yields 2^{20} possible model structures. Indeed, for two variables (here R and P), equations can have 10 terms each, which makes 2^{10} each, and then 2^{20} for the two equations. The GPoM algorithm includes two stages: a first stage makes a drastic structure pre-selection, but without fostering smaller or larger equation sizes. On the contrary, it keeps a full range of possibilities for each equation, that is, from all the terms (here 10 terms) down to no term at all, which gives here eleven possible structure for each equation, and therefore $11 \times 11 = 121$ possibilities for the two equations taken together. The robustness of these preselected models are then tested by numerical integrability in the second stage."*). All is now explained more explicitly and is now consistent, thank you.

L 259 what does non-autonomous mean?

This was not explained, sorry for that. It is now done, thank you.

Section 3.4 on validation

Better justify rejected catchments eg First, models with obviously poor-performance were discarded through visual comparison of predicted and observed runoff.

This was unclear, indeed. Actually, what we did was computing the errors to select the best model for each basin. Now this has been corrected in section 3.4 (*"First, we calculated the correlation coefficient between data and predictions, as well as the RMSE for each model. Second, we selected the model with better performance for each basin (the individual model with better performance for basin 1, the individual model with better performance for basin 2, etc.; the generalized model with better performance for basins 1 to 4 etc.)"*).

Thank you.

Validation was mentioned earlier in terms of catchments left to test after calibration but here validation seems only to be about independent periods. Please clarify.

This has been clarified in section 3.4 and the new section 3.5, where these cases are now explained separately.

How was preprocessing of runoff done as mentioned in the quotes below, rather than leave the reader to chase the reference? Justify method and errors in doing so. “Because rainfall occurs mainly in the upstream portions of the basins, and runoff was preprocessed to represent discharge at the river mouth rather than at the gauge station (Rau et al., 2019)”

We agree; this fragment was not specific enough, we have brought additional details to explain it carefully (*“One specific difficulty when comparing the runoff of different basins comes from the fact that, even when the basins are equipped, the gauges are not necessarily available at the river mouth. The runoff dataset used in this study, also derived from Rau et al. (2019) enables to overcome or at least to reduce this difficulty. It consists of 49 monthly time series of runoff covering 1970-2019 (Fig. S4-S6) that are a byproduct of the conceptual GR2M model (Mouelhi et al. 2006), that is, a two-parameter monthly water balance model. Its parameter calibration was performed considering seven basins on the periods 1970-2005 (B01, B16), 1970-2006 (B32, B43) and 1970-2008 (B09, B36, B45). Its parameterization was then generalized considering three characteristic parameters: the basin area, the main channel length and the basin perimeter. The runoff was then estimated at the river mouth of each of the 49 basins by running the model using these local parameters.”*).

Thank you for this useful suggestion.

Section 3. On simulation. It is ok to do simulation but results cannot be tested, so better to focus on validation on independent catchments and periods.

We agree that validation is important, but it cannot help understanding how the model, and the hydrologic system work. From this point of view, we believe this part can really be useful because it contributes to better understand the watershed dynamics. To facilitate its reading, we have put the experiments description in Appendix, and then explain the obtained results in the discussion. We hope this helps understanding the dynamics and illustrate how the model can be useful.

Refs missing, some out of order also.

Thank you, and sorry for that. All the references have been checked carefully now.

We would like to thank again the reviewer for his careful reading and his valuable concerns that helped us to improve the manuscript significantly.

***A new class of monthly rainfall-runoff models for ungauged basins –
Application to the Peruvian coastal basins***

Running title: A new class of monthly rainfall-runoff model for ungauged basins

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Abstract:

Rainfall–runoff modelling in data-scarce regions remains a major challenge in climatology due to limited hydrometric records and the heterogeneity of watershed responses. To address this problem, we introduce a new class of parsimonious nonlinear rainfall–runoff models that rely solely on two geometric descriptors of the basin: surface area and elongation ratio. The models are derived using the global modeling technique, insensitive to the initial conditions. Models are built and validated across 49 basins distributed over ~223,000 km² in the Peruvian Pacific slope and coast. Two generalized formulations were obtained: Model G_I , based on the four northernmost basins, and Model G_{II} , derived from a larger set. Both reproduce key hydrological features, including annual maxima, minima, and the geometry of phase-space orbits. Model G_{II} , although less parsimonious, consistently outperforms G_I in terms of error metrics, while retaining physical interpretability.

Simulations reveal that elongation exerts a stronger influence on runoff dynamics than area, particularly under extreme rainfall (e.g. El Niño events). Stability analysis further shows that basins with intermediate elongation (~0.45–0.60) are the most robust, whereas very rounded or highly elongated basins approach instability or divergence. Importantly, the proposed framework is transferable to ungauged basins, offering predictive capability with minimal parameterization.

These findings highlight the structural similarity of watershed responses across diverse climatic and geomorphic settings in coastal Peru, suggesting broader applicability to other semi-arid regions with episodic extreme events.

Keywords: basins runoff, global modelling technique, rainfall-runoff modelling, data-driven approach, ungauged basins

1. Introduction

Hydrological dynamics rely on numerous factors, both endogenous (such as topography, soil properties, surface vegetation, subsurface and underground hydrologic network, etc.) and exogenous (e.g., climatic forcing, groundwater pumping, irrigation, etc.). To generate accurate simulations of the hydrological dynamics requires detailed monitoring, which is not always possible or feasible.

Among the hydrological variables, runoff is of particular interest because it provides integrated information about the watershed dynamics as a whole. When both precipitation (typically the main input) and runoff are available, rainfall-runoff models can be very useful to get an overview of the other

contributions, particularly water use. Moreover, runoff is a key variable for water resources management and flood risk assessment.

Several types of monthly [rainfall-runoff](#) models have been developed for this purpose. On the one hand, mechanistic and conceptual [rainfall-runoff](#) models such GR2M, [GR5M](#) ([Mouelhi et al., 2006](#)), AWMW ([Boughton, 2004](#)), ABCD ([Thomas, 1981](#)), [XM](#) ([Xiong and Guo, 1999](#)), [DWBM](#) ([Zhang et al., 2008](#)) or the [VUB](#) ([Vandewiele and Ni-Lar-Win, 2009](#)) aim to represent the main or dominant processes at the basin scale. These approaches assume that the model structure is known and only require parameter calibration, which can be achieved from field observations. On the other hand, black box approaches (including Artificial Neural Networks, Support Vector Machine, and others) make few assumptions about system behavior. While highly versatile, these approaches often yield models that are difficult to interpret, as their ad hoc formulations are no more transparent than the original system ([Riad et al., 2004](#); [Raghavendra and Deka, 2014](#); [Stefanoiu et al., 2024](#), among others). Meanwhile, alternative approaches have been developed under the assumption that, although the model structure is initially unknown, it can be meaningfully reconstructed. This can be achieved through libraries of structures associated with optimization techniques. For instance, libraries that incorporate a wide range of potential hydrological processes ([Craig et al., 2020](#); [Spieler et al., 2020](#)) have been employed to rebuild model structures.

However, in many cases, data scarcity poses a critical challenge, and sometimes even runoff records are unavailable. This raises important questions regarding the generality and transferability of hydrological models when only limited information about watershed properties exists. To address this issue, alternative methods have been developed for ungauged basins, which represent a long-standing ([Pickup, 1977](#)) and challenging problem ([Sivapalan, 2003](#); [Wagener, 2004](#); [Westerberg, 2016](#)). For example, [Castiglioni et al. \(2009; 2010\)](#) used the multiple regression technique to relate rainfall with runoff in order to estimate the runoff in an ungauged basin, taking in account the drainage area, the percentage of permeable area, a synthetic description of the topography, the main channel length, the time of response, and the mean annual precipitation and temperature.

In this work, we focus on runoff and its relationship with the rainfall in 49 basins located along the Peruvian coastal slope and lowlands. Rainfall dynamics in South American coastal basins are governed by large-scale processes involving non-linear atmosphere-ocean interactions in the Pacific and at the global scale. These dynamics can produce both extreme rainfall excesses and deficits, with profound socio-economic impacts, particularly on agriculture and water security ([Rodríguez-Morata et al., 2018](#); [Moquet et al., 2018](#); [Rau et al., 2017](#)). Moreover, their high degree of unpredictability complicates modelling efforts.

Rau et al. (2017, 2019) provided a comprehensive analysis of climate and hydrology in 49 Peruvian coastal basins. Using statistical and wavelet analyses, they classified the region into nine climatic zones and compiled an extensive hydrometeorological dataset covering 1970–2019, which serves as the foundation for our work.

Here, we apply the global modelling technique for equation extraction. This mathematical method is capable of detecting nonlinear relationships without requiring strong a priori assumptions about model structure, parameterization, or underlying processes (Mangiarotti et al. 2025). Typically, it does not account for spatial variability in parameters. In our case, the method is applied to runoff dynamics at the basin scale. First, we derive nonautonomous Ordinary Differential Equations (ODEs) for each of the 49 watersheds. Next, we attempt a geographical generalization by incorporating basin area and shape, testing the approach on a subset of catchments before extending it to the full set. Our approach enables the estimation of runoff in ungauged basins using only precipitation, basin area, and basin elongation as input variables, whether derived from observed data or hypothetical scenarios. It should be noted that model parameterization requires only two parameters that can be directly derived from a Digital Terrain Model (DTM) of the basin, and the models are also computationally efficient. Furthermore, they enable the interpretation of hydrological processes a posteriori and the analysis of how these variables influence runoff across the study area.

2. Study Sites and Data

2.1. Region of Study and Watersheds

This study considers 49 distinct watersheds located along the Pacific slope of Peru (Figure 1). Each watershed extends from the Andean foothills to the Pacific Ocean, forming a continuous sequence from North to South. For reference, we label them B01, B02, ..., B49. These basins span a pronounced latitudinal climatic gradient: mean annual precipitation reaches 594.8 mm in the north (B01, Piura) but only 123.9 mm in the south (B49). Rainfall is concentrated in the mountains, and many rivers often dry before reaching the coast, even during winter.

In this region, two problems are particularly striking: droughts in the northern highlands during strong La Niña years, and floods in the middle and southern lowlands during strong El Niño years. El Niño and La Niña refer to two very specific situations of the coupling between ocean and atmosphere in the Pacific Ocean. El Niño situation occurs when surface temperatures in the eastern Pacific Ocean are warmer than average, shifting rainfall eastward toward South America, including Peru. Conversely, La Niña events push rainfall westward toward Asia, often resulting in drought across parts of Peru (Lavado-Casimiro et al. 2013). El Niño is a complex and irregularly periodic phenomenon whose pseudo-periodicity is variable

and highly unpredictable [in terms of hydrological impact on the South American continent](#) (Bourrel et al., 2015). During strong events, it couples with the annual continental cycle, producing heavy summer rains (Sanabria et al., 2018). However, the magnitude of El Niño does not always correspond directly to rainfall intensity at all latitudes (Lagos et al., 2008; Bourrel et al., 2015), reflecting a nonlinear atmosphere–ocean coupling.

Basin morphology also strongly influences the drainage system, and consequently, the runoff response (Sassolas-Serrayet et al., 2018; Rau et al., 2019). The relationship between rains and runoff can be described using the elongation ratio p_e (Sassolas-Serrayet et al., 2018). Based on this metric, we classified the 49 basins as follows: 24 are very elongated ($p_e < 0.5$), 23 are elongated (0.5–0.7), one is less elongated (0.7–0.8), one is oval (0.8–0.9); none approach the circular category (0.9–1.0). The most elongated basin is Olmos (B3), while the least elongated is Chilca (B27), and the only oval basin is Grande (B35). The drainage networks are generally symmetric, with some exceptions: the main stream lies near the northern boundary in B01, B34, and B35, and near the southern boundary in B45 and B47.

Basin size is another key factor, although the runoff–area relationship in dry lands is often complex (Tooth 2000; Reid et al. 2011). In our dataset, basin areas span nearly six orders of magnitude, ranging by a factor of ~28: from 620.3 km² in the Topara basin (B31) to 17,152.8 km² for Camana basin (B43), with a median of 3,062.6 km², corresponding to Chancay Huaral (B23) (Table 1). In this study, for clarity, we classified the basins into five size groups: very large (>15,000 km², [two basins](#)), large (10,000–15,000 km², [two basins also](#)), medium (5,000–10,000 km², [three basins](#)), small (2,500–5,000 km², [16 basins](#)), and very small (<2,500 km², [23 basins](#)). Larger and medium-sized basins are concentrated in the South, with the exception of Piura (B1) in the North and Santa (B13) in the central region. The smallest basins tend to be shorter, reflecting rivers that originate closer to the coast (e.g., Omas). Overall, basin area is roughly proportional to river length, except B35, B42, and B44, which are anomalously wide compared to their length.

2.2. Rainfall Dataset

The rainfall dataset consists of one representative time series per basin, coming from Rau et al. (2019), which yields a total of 49 time series with monthly [data](#) spanning 1970–2019, [here extended to 1970–2023](#). These time series were constructed by Rau et al. (2019) to represent basin-wide precipitation, based on [in-situ](#) measurements from multiple stations within each basin and subsequently preprocessed to obtain a single representative record (Fig. S1–S3). For simplicity, we denote precipitation as ϖ , and its square root as P .

Minimum precipitation is close to zero in all basins (0.1–0.3 mm), reflecting dry-season conditions (Table S1). In contrast, median precipitation varies markedly among basins: Santa (B13), Chancay

Lambayeque (B05), and Jequetepeque (B08) are the wettest, with median values exceeding 45 mm a year. Nine basins show medians between 20 and 45 mm, the southernmost of which is Huaura (B22), while the remaining basins record medians below 20 mm. During extreme El Niño events, the basins with the highest maxima are not necessarily those with the largest medians. The clearest example is Piura (B1) (>500 mm), which has a relatively low median (14.4 mm) but records maxima above 500 mm, reflecting its strong sensitivity to El Niño. Eight other basins, all in the north (B02-B06, B08, B09 and B13), show maxima between 350 and 500 mm. In general, rainfall magnitude decreases from north to south, though exceptions exist—for instance, Ocona (B42) is relatively wet despite its latitude. Rainfall also tends to decrease from the mountains toward the coast, but since our analysis uses basin-averaged values, this gradient is not explicitly considered.

2.3. Runoff Dataset

One specific difficulty when comparing the runoff of different basins comes from the fact that, even when the basins are equipped, the gauges are not necessarily available at the river mouth. The runoff dataset used in this study, also derived from [Rau et al. \(2019\)](#) enables to overcome or at least to reduce this difficulty. It consists of 49 monthly time series of runoff covering 1970-2019 here extended to 1970-2023 (Fig. S4-S6) that are a byproduct of the conceptual GR2M model (Mouelhi et al. 2006), that is, a two-parameter monthly water balance model. Its parameter calibration was performed considering seven basins on the periods 1970-2005 (B01, B16), 1970-2006 (B32, B43) and 1970-2008 (B09, B36, B45). Its parameterization was then generalized considering three characteristic parameters: the basin area, the main channel length and the basin perimeter. The runoff was then estimated at the river mouth of each of the 49 basins by running the model using these local parameters. For this reason, this product represents a spatially and temporally extended version of the original records, recalibrated and validated against in-situ hydrometric stations to estimate runoff at each river mouth, regardless of the gauging station location. This approach facilitates direct comparison among basins. Anthropogenic influences were minimized by using only semi-natural gauged basins as signals for calibration and validation (see [Rau et al, 2018](#)). For simplicity, runoff is denoted as ρ , and its square root as R .

Although monthly precipitation is generally scarce in the study area, minimum values are usually above zero ([Table S1](#)). In contrast, median runoff at the river mouths is often zero or near zero. Only a few basins show higher median runoff: Santa (B13, 13.8 mm), Ocona (B42, 8.4 mm), and Chancay Lambayeque (B05, 6.2 mm). These basins, along with Pativilca (B20) and Cañete (B30), also display relatively high third quartiles (15-45 mm). The remaining basins exhibit much lower values, with medians below 3.5 mm and third quartiles between 5-15 mm (19 basins) or 0-5 mm (25 basins).

The influence of El Niño on basin hydrology is clearly expressed in the runoff maxima. The Piura basin (B01) records the highest maximum (469.8 mm), consistent with its strong sensitivity to ENSO-driven rainfall. The next largest maxima are observed in Santa (B13) and Chancay Lambayeque (B05), both around 275 mm. Several other basins—B02, B04, B06, B08, B09, B10, as well as Pativilca (B20), Cañete (B30), and Ocoña (B42)—exhibit maxima between 175 and 250 mm, the latter three being notable given their more southerly location. In general, runoff tends to decrease from north to south, though the pattern is not systematic. Unlike precipitation alone, runoff variability reflects the combined influence of rainfall, basin geometry, evapotranspiration, and other hydrological processes, resulting in irregular spatial distributions.

2.4. Pre-processing

While higher-resolution data could provide additional detail, such data are not available for this large temporal and spatial extent, and their inclusion would likely increase model complexity at the expense of interpretability. Therefore, the original monthly time series were upsampled to 12 values per month. Without this refinement, the coarse timestep may hinder the estimation of the derivatives and compromise the reconstruction. Upsampling also enhanced the ability to capture nonlinearities of the system. We employed Stineman interpolation (Stineman, 1980), which minimizes the number of inflection points and, thus, avoids the artifacts commonly introduced by spline interpolation (Zhang and Martin, 1997). The procedure occasionally generated small negative values, which were subsequently set to zero.

No additional smoothing was required beyond the implicit filtering performed by the global modelling code during the estimation of the derivatives. This algorithm uses a default window of nine timesteps (equivalent to approximately three weeks), whose effect on the data was marginal. It was necessary to apply a square root transformation to the data, as searches using the original variables produced no results—likely due to the large disparity between dry and wet years. We actually tested three approaches—log transformation, no transformation, and square root transformation—but only the square root transformation produced numerically stable models. All models were therefore constructed using the transformed variables ($R = \rho^{0.5}$ and $P = \varpi^{0.5}$). Model errors, however, were evaluated before and after back-transforming to the original runoff variable (i). The errors of the squared runoff are more representative of the low-flow season, while the original runoff better represents high-flow events. For visualization, the root-transformed series (R and P) were occasionally retained.

Finally, the time series were divided into two periods: from 1 January 1970 to 10 April 2003 for model calibration, and from 12 April 2003 to 1 December 2019 for independent validation.

3. Theoretical Background

This study focuses on two main variables: runoff (ρ), which is the primary variable of interest, and rainfall (ϖ), considered as the main forcing. Although causally related, these variables exhibit distinct dynamics. Rainfall is governed by deterministic physical processes but is characterized by high dimensionality and stochastic-like variability (von Storch, 2004). Runoff, by contrast, reflects the integrated effect of rainfall over the basin and thus displays smoother, lower-dimensional dynamics (Mangiarotti et al., 2019). Because the influence of the El Niño phenomenon is already embedded in precipitation records, including sea surface temperature in the models offered no added predictive value (on the contrary, it only increased complexity).

3.1. The Global Modelling Technique

The global modelling technique takes its background from the theory of nonlinear dynamical systems, or theory of chaos (Bergé et al. 1984). This theory enables to understand how a system can be both deterministic and unpredictable in the long term, due to high sensitivity to initial conditions. The global modelling takes advantage of this theory by developing a technique independent to the initial conditions. It is designed to extract dynamical equations directly from time series and also applies to chaotic dynamics (Aguirre and Billings, 1995; Gouesbet and Letellier, 1994). Initially tested to theoretical cases, it was successfully applied in the early 1990s to single experimental time series in electrochemistry and mixing reactors (Letellier et al., 1995; Letellier et al., 1997). More recently, it has been extended to multiple observational time series from real-world systems (Mangiarotti and Huc 2019, Mangiarotti 2015). The approach has also been shown to handle multiple time series taken in association (Mangiarotti et al., 2016) and to model the dynamics of non-autonomous systems (Sáez et al., 2021; 2024).

In this work, two formulations are considered. The first is a local (ad hoc) formulation:

$$dR_r/dt = F(R_r, U_r(t)) + \varepsilon \tag{1}$$

where R_r denotes the modelled variable (here, the square root of runoff) and $U_r(t)$ represents the forcing (here, the square root of precipitation, P), explicitly dependent on time t , for the basin r , centred at coordinates (λ, φ) . F is a function and ε a dynamical noise assumed to be Gaussian and centred. The second is a generalized formulation

$$dR_r/dt = F(R_r, U_r(t), p_a, p_e) + \varepsilon \tag{2}$$

where p_a and p_e are basin-specific geographical parameters corresponding to area and elongation ratio, respectively. This new formulation aims to generalize the usual global modelling technique. In most of the previous studies, this technique was applied locally, or, at best, assuming that the spatial variability could be resumed as a single dynamic, either by aggregating the observations as a homogeneous signal, or by considering the time series in association, that is, as an ensemble of dynamics possibly independent (in phase, in initial conditions) but all identical (same equations and parameterizations). This problem was investigated both theoretically by considering ensembles of Rössler systems (Mangiarotti et al. 2016) or practically for the cycles of cereal crops observed by satellite in Morocco (Mangiarotti and Le Jean 2023). The formulation presented here in Eq. (2) is still independent from the initial conditions and the forcing, but more general by reconstructing not only the original set of equations but also parameter dependence. This is what makes this approach a new class of global models because it aims at reproducing not only the global dynamics as a particular case, but as general case taking also in account the variations of its parameters. Of course, only the role of the parameters provided as an input can be expected to be involved in the reconstructed set of equations. The parameters p_a and p_e are incorporated into the generalized formulation (Eq. 2) to model multiple basins simultaneously, whereas the local formulation (Eq. 1) targets individual basins and does not include p_a or p_e explicitly.

In practice, the GPoM R package was used to obtain a polynomial approximation of the function F , both in terms of structure and parameterization, following the procedure presented in Mangiarotti and Huc (2019), with adaptations to account for multiple basins when constructing generalized models. The same modelling period was applied to all the time series, spanning from 1 January 1970 to 10 April 2003. All analyses were performed using the GPoM package (Mangiarotti et al., 2023), which is freely available from the Comprehensive R Archive Network (CRAN): <https://CRAN.R-project.org/package=GPoM>.

3.2. Adaptations to the Study of Generalized Models

Some methodological modifications were introduced to derive generalized models, i.e., single formulation models applicable to multiple basins. These models capture broader dynamics, as they are derived from data collected from multiple basins. To achieve this, time series for all basins involved in the modelling were considered simultaneously in the modelling process. To do so, the time series of each variable were concatenated sequentially to create a single extended precipitation time series. Basically, concatenation was performed by assembling the time series of the modelling basins one after the other, as illustrated in Figure 2 (here considering four modelling basins). In the present case, we need four time series: the first one for $R(t)$, the second one for $P(t)$, the third one for $p_a(t)$ and the fourth one for $p_e(t)$. This way, each variable includes the concatenated information of the four basins used for the modelling (B01, B02, B03 and B04, respectively in purple, pink, cyan and blue colours). For each basin, the area

and the elongation parameters are constant in time, but they vary spatially from one basin to another. A gap was inserted between each consecutive basin series in order to avoid spurious values in the derivatives. The combined time series were used as input for the GPoM function. Model validation and error estimation were performed independently for each basin, using the validation periods, before and after reversing the square-root transformation.

First, a generalized model (G_I) was developed using the first four basins (Piura to Motupe) which differ considerably in size and shape but are relatively dry on average (see Table S1). This model was found applicable to most of the 49 basins, with varying but generally acceptable performance.

To improve representativeness, a second generalized model (G_{II}) was then constructed using basins distributed across all latitudes of the study region. Specifically, the modelling process was applied to every third basin in the sequence—i.e., the 1st, 4th, 7th, ... up to the 49th—resulting in a set of 17 basins. This choice was applied systematically, no specific basin selection was operated a posteriori in the modelling process. The remaining 32 basins were reserved for validation.

Note that the modelling and validation periods, and the modelling and validation basins are all synthesized in Table 2. There was no ad hoc decision to include or exclude any catchments: these choices were applied systematically and were not changed in the modelling-validation process.

3.3. Adaptations to Handle the Number of Possible Models

In the original GPoM algorithm, the model search for a two-dimensional system (one equation for the runoff and one for the precipitation) with maximum polynomial degree of three, yields 2^{20} possible model structures. Indeed, for two variables (here R and P), equations can have 10 terms each, which makes 2^{10} each, and then 2^{20} for the two equations. The GPoM algorithm includes two stages: a first stage makes a drastic structure pre-selection, but without fostering smaller or larger equation sizes. On the contrary, it keeps a full range of possibilities for each equation, that is, from all the terms (here 10 terms) down to no term at all, which gives here eleven possible structure for each equation, and therefore $11 \times 11 = 121$ possibilities for the two equations taken together. The robustness of these preselected models are then tested by numerical integrability in the second stage.

In the present study, however, we deal with a non-autonomous equation for the root runoff (Eq. 1), that is, an equation explicitly forced by an external variable $P(t)$ (the root precipitation). Thus, it is neither necessary nor feasible to model an equation for precipitation with a high precision: its dynamic is high-dimensional, while GPoM is designed for low-dimensional systems. To account for this, we adapted the code to restrict the search to the runoff equation only. This modification substantially reduced the number of possible model structures to be tested ($N = 2^{10}$ for a single equation of two variables, here P and R , with maximum polynomial degree of 3).

For the generalized models, the number of possible model structures increases substantially because four [variables](#) are considered instead of two, which also raises the maximum polynomial degree from 3 to 5. This expansion results in $N^* = 2^{126}$ possible model structures, compared to N for the individual models. [Indeed, although](#) the watershed geometric parameters (p_a and p_e) are ultimately used as coefficients, they must be introduced during model search as stepped time series ($p_a(t)$ and $p_e(t)$) together with $R(t)$ and $P(t)$. In the individual models, the maximum polynomial degree was limited to 3. In the generalized case, however, each term may also be multiplied by p_a , p_a^2 , p_e , p_e^2 and $p_e p_a$. Since the maximum polynomial degree of R and P combined is 3 and the maximum polynomial degree of p_a and p_e combined is 2, the total maximum polynomial degree reaches 5. For example, the term RP^2 expands into six possible variants: RP^2 , $p_a RP^2$, $p_a^2 RP^2$, $p_e RP^2$, $p_e^2 RP^2$, and $p_a p_e RP^2$.

Finally, we applied a template derived from the individual models, discarding all the terms that did not appear in any of them.

3.4. Validation

[At this point, each model search retrieves multiple individual models based on the same template and it is necessary to select the best for each basin.](#) The models were validated by numerical integration over the validation window, using the observed root precipitation as forcing. The same validation period was applied to all time series, from April 12, 2003, to December 1, 2019. First, we calculated the correlation coefficient between [data](#) and predictions, as well as the RMSE [for each model. Second, we selected the model with better performance for each basin \(the individual model with better performance for basin 1, the individual model with better performance for basin 2, etc.; the generalized model with better performance for basins 1 to 4 etc.\)](#)

3.5 Application to other basins and comparison

These global error measurements, however, have some limitations [when it comes to comparing the performances of either multiple models on the same basin or a single model on multiple basins.](#) Because the original variables were squared for modelling, the errors tend to increase as the runoff values become larger. Thus, a reasonable RMSE for an extreme event (e.g., 10 mm) can exceed the annual maximum runoff in a year of scarce precipitation, even within the same basin. Similarly, accurate reproduction of extreme events may yield an overly optimistic correlation coefficient. To better assess model performance across different hydrological conditions, these errors were computed for each hydrological cycle (September–August). The statistics of the annual RMSEs were, then, derived for each basin and model. As noted above, RMSE generally increases with runoff maxima. To normalize this effect and enable inter-basin comparison, we defined a relative maximum RMSE, obtained by dividing the maximum RMSE by

the maximum observed runoff. This metric provides a measure of the relative performance of each model during extreme events. Finally, we also analyzed the RMSE calculated directly from the root square runoff in order to get an error that is less sensitive to the high events and more appropriate to assess low-flow and intermediate-flow conditions.

4. Results

Because rainfall occurs mainly in the upper watershed, runoff data were preprocessed to reflect conditions at the basin outlet rather than at the mid-basin gauge stations. This adjustment was performed using a regional hydrological model based on GR2M (Mouelhi et al., 2006), which aggregated precipitation across the entire basin (Rau et al., 2019). Therefore, our global models capture rainfall forcing and its downstream response—after losses by infiltration, evaporation, and other processes. In fact, in many basins and years, rivers may dry out completely before reaching the ocean. Anthropogenic influences are, in principle, excluded thanks to the preprocessing procedure of Rau et al. (2018).

4.1. Individual Models

An individual model was obtained for each of the 49 basins, resulting in 19 different model structures (Table 3), labelled from **a** to **s**. None of the individual models includes a constant term or a linear precipitation (P) term; these are, therefore, omitted in Table 3. Structures **a**, **d**, **f**, **g**, **h**, **i**, **j**, **k**, **l**, **n**, **o**, and **r** (39 models) are all mutually compatible in their structure: terms may be present or absent, but shared terms always have the same sign. In contrast, structures **e**, **m** and **p**, (6 models) are incompatible with the previous group, as they contain positive RP and R^2P terms. Structures **q** and **s**, corresponding to two southernmost basins (models for B47 and B49), are unique, and compatible with each other. Finally, structures **b** and **c** (respectively for B03 and B04 models) are unique, but **c** is compatible with both **b** and the **e-m-p** group, whereas **b** is not (see Table 3 caption for details).

For the individual models, global RMSE values on the validation period (2003-2023) range from 0.08 mm (B27) to 10.34 mm (B01), with a median of 3.02 mm (Table S2). These values are generally lower than those of the generalized models, since the calibration focused specifically on each basin. Correlation coefficients vary between 0.82 (B46) and 0.98 (B05), with a median of 0.96.

Analysis of the RMSE across hydrological cycles yields further insight into model accuracy (Table S3). Piura basin (B01) exhibits the highest 90% percentile RMSE (19.63 mm) and the second highest maximum RMSE (25.88 mm). However, because this basin experiences the largest runoff maxima due to the strong El Niño influence, its relative maximum RMSE is only 5.5%, indicating highly accurate

predictions (Figure 3). In contrast, Chilca (B27) shows the lowest maximum RMSE (0.28 mm). Despite a small maximum runoff of 1.78 mm, the relative maximum RMSE is 15.6%. Nevertheless, the model accurately predicts the extreme dryness and minimal runoff of this basin (Figure 3). These examples highlight the importance of considering both absolute and relative RMSE when evaluating model performance.

The lowest-performing individual models correspond to Motupe (B04) and Ocona (B42). Motupe (B04) exhibits the highest maximum RMSE (35.94mm) with moderate maxima, resulting in a relative maximum RMSE of 15.3%. Despite this, the model successfully distinguishes between low, medium, and extreme events (Figure 3). Ocona (B42), a large Southern basin, ranks third in 90% percentile RMSE (11.46 mm) and seventh in maximum RMSE (12.05 mm), with a maximum runoff of 84 mm. Its relative maximum RMSE is 14.2%, indicating moderate performance, with some overestimation of a few annual maxima (Figure 3). The largest relative error is observed in B39, at 22.2% (Figure 3).

The RMSE computed from the root precipitation P ranges from $0.08 \text{ mm}^{0.5}$ for B27 to $0.92 \text{ mm}^{0.5}$ for B01. Basin B35 RMSE represents only the 3.3% of the maximum recorded root runoff (during the validation period) and basin B42, the 10.02%. Overall, the results indicate good performance in absolute and relative terms.

4.2. Generalized Models G_I

Model G_I was derived using only the first four basins – the most northern ones (B01 to B04) – which differ considerably in size and shape. Despite these differences, in latitude, shape and size, the generalized model obtained from the four northern basins performs well across most of the 49 basins, indicating that their general hydrodynamic structure is compatible. The generalized model

$$\dot{R} = (\theta_1 p_e - \theta_2 p_e^2) R P^2 - \theta_3 R^2 - (\theta_4 - \theta_5 p_e) R^2 P \quad (3)$$

is parsimonious, comprising five terms that reduce to three when the elongation ratio p_e is fixed. Notably, the area parameter p_a was not retained (although it was included in the search), indicating that basin shape (elongation) is more influential than size within this group of four basins. Structurally, generalized models resemble the individual structure **a**, with terms in RP^2 , R^2 , and R^2P , but it also incorporates information about basin shape, allowing adaptability to basins whose individual structures differ (e.g., Olmos and Motupe, corresponding to structures **b** and **c**).

The coefficient of RP^2 is always positive for elongation ratios between 0.3 and 0.674, typical of most basins except the more rounded ones: Chilca (B27) and Grande (B35). The R^2P coefficient is negative for basins with elongation ratios lower than 0.546 (that is, for basins B01-B03, B05, B06, B08, B09, B13,

B14, B18, B20-B22, B24-B26, B28, B30-B34, B36, B39, B41, B43, B45-B47 and B49), that is, almost two thirds of the basins. Finally, the R^2 term is consistently negative and independent of basin geometry.

Overall, G_I can be applied to almost [all the](#) basins, with varying performance. Remarkably, a model based on four northern basins approximates the dynamics of basins across the entire study area, despite the latitudinal and climatic gradient. Performance is more irregular than for individual models, reflecting the limited calibration base. Notably, the model ignores the basin area, as it was rejected by the algorithm despite large size differences among the four basins.

[Figure 4](#) plots each basin maximum RMSE against its relative maximum RMSE, with the line $y = -0.002x + 0.281$ (where x is the maximum RMSE and y is the relative maximum RMSE), separating poor (above the line) from acceptable performance (below). Some large southern basins (B35, B37, B38, B42 and B44) exceed 30% relative maximum RMSE, making the model G_I unsuitable. Certain small middle basins (B12, B15, B16, B23) also perform poorly (maximum RMSE >25 mm; relative RMSE >25%). In some very dry basins, the maxima are underestimated but the overall performance is moderate because their maximum RMSE is still low. For the rest of the basins, combinations of absolute and relative maximum errors allow the application of model G_I . For example, most basins with considerable runoff – such as those located in the north and under the direct influence of El Niño (B01-B03, B05, B06, B08, B09) – have maximum RMSE between 15-35mm but their relative maximum error is always under 10%. Very dry basins (such as B39, B40, B48 and B49) have considerable relative maximum RMSE but still very low maximum. Finally, the basins with better performance (maximum RMSE <12mm and relative maximum RMSE <12%) are B03, B08, B21, B28 and B45; these basins are limited neither to the modelling basins nor to the North, instead they cover the whole latitudinal range.

These results are confirmed by the RMSE computed from the root runoff, which ranges from 0.20 mm^{0.5} for B27 to 2.29 mm^{0.5} for B16. For a few basins (B12, B15, B16, B23, B35 and B42, already discarded in the previous paragraph) the RMSE is equal to or larger than 1.5 mm^{0.5} and all of them correspond to basins with low flow where the model G_I fails to reproduce the maxima.

4.3. Generalized Model G_{II}

The generalized model G_{II} (Eq. 4)

$$\begin{aligned} \dot{R} = & \beta_0 R P^2 + R^2 (\beta_1 - \beta_2 p_e + \beta_3 p_e^2) - R^2 P (\beta_4 - \beta_5 p_e + \beta_6 p_e^2 - \beta_7 p_a + \beta_8 p_e p_a + \beta_9 p_a^2) \\ & + R^3 (\beta_{10} - \beta_{11} p_e + \beta_{12} p_e^2 - \beta_{13} p_a + \beta_{14} p_e p_a + \beta_{15} p_a^2) \end{aligned} \quad (4)$$

was derived from basins B01, B04, B07... to B49. Compared to G_I , it is less parsimonious, as it includes the area p_a , but it achieved lower overall errors (see [Table S2-S3](#)). The model contains sixteen terms,

which reduce to only four once p_a and p_e are specified for a given basin. The resulting simplified structure corresponds to structure **a**, with the addition of an R^3 term (see Table 3).

Model G_{II} can be applied to most basins, though with some exceptions. Predictions diverge for basins B03, B27, B35, and B49 (hence they are absent from Figure 4B). Moreover, for basins B42 and B44, the annual maxima are strongly underestimated—similar to the behaviour of model G_I —while for B41, the model fails to reproduce the annual seasonality correctly. Basins B17, B34, and B40 show only moderate performance: they present high relative maximum RMSEs ($>25\%$) but low absolute maximum RMSEs (<15 mm). For the remaining basins, however, model performance is satisfactory. The RMSE calculated from the root runoff ranges from $0.24 \text{ mm}^{0.5}$ for B48 to $2.09 \text{ mm}^{0.5}$ for B42. The latter, along with B41, has a root runoff RMSE larger than $1.5 \text{ mm}^{0.5}$ confirming the problems mentioned above for these basins. Overall, model G_{II} outperforms G_I in terms of absolute RMSEs, as illustrated in Figure 4, where the cloud of points in the right panel (G_{II}) shift clearly to the left compared to the left panel (G_I). Relative maximum errors are also lower for G_{II} , with fewer basins located in the upper part of the right-hand plot. Although a large portion of the diagram remains empty, the same reference line used for G_I can still be applied here, effectively distinguishing poor (orange and red) from good (green) performance. Only two basins, B38 and B39, fall above this line. In the green-performance area, the only basins with maximum RMSE exceeding 25 mm are B01 and B05, yet both retain relative maximum RMSEs below 10%. This outcome is expected, as these northern basins experienced strong maxima during the El Niño event included in the validation window.

4.4. Phase portraits

The phase portrait projections of precipitation versus runoff describe anticlockwise, lance-shaped orbits (Figure 5A-5B). Smaller orbits are more horizontal, while the larger ones more oblique. For medium to high events, three distinct slopes emerge as precipitation and runoff increase. At low precipitation (<25 -50 mm, corresponding to 5 - $7.5 \text{ mm}^{1/2}$ in the plots), runoff remains close to zero, producing a nearly flat slope. The second slope is clearly positive but moderate, while the third is steeper, marking the sharpest runoff response. After the annual maximum is reached, the orbits descend in a single stretch, though the slope sometimes changes gradually.

By construction, differential phase portraits (R , dR/dt) are clockwise. They reveal further diversity of shapes across basins. For example, basin B01 (Figure 6A, grey line) displays an approximately triangular shape with three different slopes. First, runoff increases gradually from near 0 towards its maximum (upper left side of the triangle). In the second slope, the runoff continues to raise until its peak, but the growth rate slows and may even turn slightly negative. This second slope may be gradual (as in B01), but in many basins it changes more abruptly. Finally, runoff decreases slowly, with the rate shifting from negative to

near zero, creating an almost horizontal segment along the triangle's base. Large events (larger orbits) typically produce more irregular trajectories than the frequent cycles and clearly exhibits quick runoff decreases when its values are high. By contrast, other basins such as B09 (Figure 6B, grey line) exhibit more symmetric, drop-shaped orbits with smoother transitions.

All three models (individual, G_I , and G_{II}) reproduce the overall size and shape of the orbits in the phase space with accuracy (Figure 5). The precipitation versus runoff plots (Figure 5A-5B) capture the annual maximum runoff, the descending slope, and the annual minimum reasonably well.

Differences arise, however, in the representation of the ascending slopes: the model orbits transition more smoothly than the [original data](#), producing slightly wider orbits compared to the sharp lance-shaped curves in the data. In the models, the two ascending slopes are connected by a gradual transition instead of an abrupt inflection, and the annual maximum is reached and left in a smoother manner. This behaviour is expected, since the models are based on the derivative of the runoff and, therefore, cannot reproduce the abrupt changes seen in the [data](#). A finer temporal resolution of the original rainfall and runoff records (currently one month) could potentially improve the ability of the models to capture these faster slope variations *and likely improve predictive performance, particularly for estimating extreme events. However, it would also increase model complexity, making the results more difficult to interpret. For this reason, we preferred to reserve the analysis of higher-resolution data for future work.*

5. Discussion

Most of the data-driven approaches will produce black-boxes which interpretation can be very limited. The global models here obtained have an algebraic structure which can be analyzed to interpret the hydrological processes, a posteriori. The interpretation can even go further, here, because the equations also include parameters (here the basins area and elongation) that are characteristic of the hydrological basins.

In general, the models contain three types of terms: those depending only on precipitation P (P^2 and P^3), those depending only on runoff R (R , R^2 , and R^3), and mixed terms involving both P and R (RP , R^2P , and RP^2). Terms depending solely on P capture the effect of precipitation as an external forcing, contributing to watershed recharge, and, therefore, they are expected to be positive. In contrast, terms depending only on R reflect the emptying of the watershed (an autonomous process independent of precipitation) and they are expected to be negative. Mixed terms, involving both P and R , reflect the interaction between rainfall and the current hydrological state of the watershed, showing that watershed

dynamics are not solely determined by precipitation, but also by the current state of the basin. These mixed terms can be either positive or negative.

For the individual models, the terms in P (P^2 and P^3) are consistently positive, as expected (Table 3). Terms in R and R^2 are always negative, while the term in R^3 is negative in all cases except for structures **q** (basin B47) and **s** (basin B49). For basin B47, the sum of the R terms remains, nevertheless, negative, whereas basin B49 shows an overall positive global contribution from R terms, which can be regarded as an anomaly. This specific characteristic may be the result of very local effects associated with dams or irrigation systems, whose effects might be difficult to remove because of the narrowness and relatively small size of these two basins: additional information would be required. Groundwater flows resulting from the complex geology of this area may have an influence on the contribution of runoff as a source rather than a sink. The RP^2 term, linear in R and nonlinear in P , might represent the dominant effect of precipitation and it is accordingly, positive in almost all basins, except for structure **b** (basin B03). Finally, the RP and R^2P terms can be negative (30 basins), positive (8 basins), or absent (11 basins). When both RP and R^2P are present, their signs are always consistent.

The generalized model G_I lacks terms depending solely on the external forcing, i.e., precipitation. It includes a nonlinear term in R (R^2), independent of basin geometry and consistently negative, as expected. The RP^2 term is nonlinear in the elongation ratio p_e and positive, except for the most rounded basins (B27 and B35). In these cases, G_I predictions converge to zero, correctly describing the minimal runoff of B27 but underestimating the small peaks of B35. For all other basins, RP^2 remains positive. The R^2P term, linear in p_e , captures the interaction between the internal dynamics and the external input: it contributes to the recharge in less elongated basins ($p_e > 0.546$) and to the discharge in more elongated basins ($p_e < 0.546$). The contribution of the term R^2P is smaller in magnitude compared to the other terms and sometimes it is close to zero (e.g., basins B18 and B25). As a result, the combined contribution of R^2P and RP^2 is always positive (Table S4).

Model G_{II} includes two nonlinear runoff terms (R^2 and R^3). The R^2 term is always negative, while the R^3 term is negative for nearly all basins, with the exception of B05, B12, B16, B21, B37 and B41–B44. Despite these exceptions, the combined contribution of R^2 and R^3 remains negative for all basins (except B41), as expected, since these terms represent the emptying of the watershed. The term RP^2 is consistently positive, reflecting its contribution to the watershed recharge. By contrast, the R^2P term is negative for 26 basins (B01, B02, B07–B11, B14–B20, B22, B25, B28–B32, B34, B39, B40, B46–B48) and positive for the remainder. Nevertheless, the joint effect of R^2P and RP^2 is positive for all basins except B41.

Thus, model G_{II} is coherent for 48 basins, with two terms representing the emptying of the watershed (R^2 and R^3) and two terms capturing the interaction between the external input and the internal dynamical

state of the water flow (R^2P and RP^2) (Table 4). Therefore, in both generalized structures (G_I and G_{II}), the contributions to runoff variations can be separated in two main components: a positive contribution (R^2P and RP^2) and a negative one (R^2 and R^3).

Figure 7 displays the distribution of basins according to their area and elongation ratio, classified into several groups. The largest group (A, cyan) corresponds to basins with intermediate elongation ratios ($\sim 0.45-0.60$) and areas up to $12,000 \text{ km}^2$ approximately. In this group, the positive RP^2 term is dominant, exceeding the also positive contribution of R^2P , while the negative effect of R^2 is stronger than that of R^3 , although all contributions remain significant (Table S5). Surrounding this cluster lies a ring of basins (group B, purple) characterized by negative R^2P values. At more extreme values of area and elongation ratio, the R^3 term becomes positive (group C, magenta), but the combined effect of R^2 and R^3 remains negative. In these basins (with the exception of B37), the dominant contributions come from R^2P and RP^2 , though R^2 and R^3 also play non-negligible roles. Finally, for basins with the most extreme values of area or elongation ratio, the models tend to become unstable (group D, orange, including B41) or even divergent (group E, red).

For model G_{II} , the R^2 term depends nonlinearly on the elongation ratio (p_e) and is independent of the basin area, whereas the R^3 term depends nonlinearly on both p_e and p_a . For these terms, either:

$$\beta_2 p_e > \beta_1 \text{ and } \beta_2 p_e > \beta_3 p_e^2 \text{ and } \beta_{11} p_e > \beta_{10}$$

or:

$$\beta_{12} p_e^2 > \beta_{13} p_a > \beta_{14} p_e p_a > \beta_{15} p_a^2.$$

The contribution of $R^3(-\beta_{13} p_a + \beta_{15} p_a^2)$ is always negative and the contribution of $R^3(\beta_{14} p_e p_a)$ is always positive, with their sum often close to zero (Table S6). Consequently, the dependency on basin area is generally secondary. Indeed, in many basins with elongation ratios between $\sim 0.5-0.6$ (B01, B04, B07, B10-12, B14, B15, B17-19, B22, B23, B25, B26, B29, B31, B38-40, B46-48), the contribution of $R^2(\beta_1 - \beta_2 p_e + \beta_3 p_e^2) + R^3(\beta_{10} - \beta_{11} p_e + \beta_{12} p_e^2)$ is clearly dominant ($\geq 90\%$) compared to $R^3(-\beta_{13} p_a + \beta_{14} p_e p_a + \beta_{15} p_a^2)$. This indicates that the basin area plays a significant role in the R^3 term only when the basin shape is considerably more rounded or elongated than average. The RP^2 term is independent of basin geometry, while the R^2P term depends nonlinearly on both p_e and p_a . For RP^2 , $\beta_5 p_e > \beta_4$ or $\beta_6 p_e^2 > \beta_7 p_a > \beta_8 p_e p_a > \beta_9 p_a^2$. The contribution of $-R^2P(-\beta_7 p_a + \beta_9 p_a^2)$ is always positive and the contribution of $-R^2P(\beta_8 p_e p_a)$ is always negative (opposite to what happened with R^3). These contributions are clearly larger than $-R^2P(\beta_4 - \beta_5 p_e + \beta_6 p_e^2)$, but they cancel each other to a great extent and their sum is of the same order as $-R^2P(\beta_4 - \beta_5 p_e + \beta_6 p_e^2)$. Sometimes they cancel each other almost completely, as in basin B01.

Simulations of model G_{II} were performed under multiple areas and elongation ratios to better understand the role of parameters p_a and p_e (see Appendix A for details). Figure 8 illustrate how the basin response differs between dry/moderate years and the extremely rainy year, when a strong El Niño event got coupled with the Peruvian hydrological cycle. As expected, smaller elongation ratios ($p_e = 0.4$ km/km) produce larger annual peaks (purple line) because the water gathered at different points of the basin reaches the river sooner. Conversely, for the more rounded basin (magenta line) the peak is smaller. During the strong El Niño event, increasing the elongation ratio from 0.51 to 0.65 km/km markedly reduced the annual maximum runoff (from ~90 mm to ~55 mm), whereas decreasing it to 0.4 km/km raised the maximum but not more than in dry and moderate years. In both cases, the hydrograph tails lengthen: in more rounded basins, the peak is smaller and water release is, thus, slower; in more elongated basins, water reaches the river rapidly, but the discharge cannot occur instantaneously, extending the tail.

The effect of basin area is less pronounced than that of the shape. Expanding the area from 4,334 to 7,000 km² produces little change in runoff, which is common in dry basins where rainfall from distant areas seldom contributes significantly to river flow. Even reducing the area to 600 km² decreases the peak only slightly. However, under the strong El Niño event, this pattern is reversed: larger basins amplify the peak, whereas reducing the area has little effect.

The analysis of the stability condition (see Appendix A.2 for details) showed that for average area and varying elongation ratios (Figure 9a), differences among basins become more pronounced as precipitation is increased. At $P^2 = 600$ mm, the runoff ranged from ~50 to ~500 mm, with most basins (80%) exceeding ~400 mm. For average shape and varying areas (Figure 9b), the response was more complex. For precipitation up to 400 mm, all basins followed similar trends. Beyond this point, differences among basins widened, with runoff ranging from 450 to 875 mm at 600 mm of precipitation. This indicates that basin area significantly influences runoff, but primarily under high precipitation conditions, which is consistent with the results of Section 5.1.

Figure 10 shows the precipitation required to sustain specific runoff levels (50, 100, 200, ... 500 mm). The white areas surrounding the colored zones indicate precipitation values above 900 mm, whilst the white tongues inside the colored zone indicate points where the discriminant is negative. In order to maintain (in a hypothetical static situation) a runoff of 50 mm, most basins require precipitations around 130 mm. However, basins B03, B35, B42, B44, and B49 require substantially higher precipitation, reflecting the known limitations of model G_{II} for these basins.

A tongue of points outside the stability range appears on the left. As the target runoff level increases, this tongue shrinks and subsequently reappears on the right, leaving a few basins (B42, B44) outside the stability range. The precipitation required to maintain higher runoff levels also increases, roughly 210 mm for 100 mm of runoff, 325 mm for 200 mm, and 600 mm for 500 mm. As the target runoff rises, more

basins—such as B27 and B41—enter the region where required precipitation escalates sharply, near the edge of the coloured zone. The slight leftward tilt of the coloured stability zone indicates that small basins are slightly more stable when they are rounded, whereas larger basins are more stable when they are elongated. Thus, the analysis of these patterns provided a basis for interpreting the underlying hydrological processes and rejecting unsuitable model structures.

6. Conclusions

This paper introduces a new class of rainfall-runoff models directly built from observational time series and basin characteristics. *These models constitute a new class of rainfall-runoff models because (1) they are global models: their aim is not to fit the time series but to reproduce the original dynamic as a whole, directly from particular time series, and independently to the initial conditions; and (2) because they enable to extract interpretable equations — including interpretable parameters characteristic of the studied (hydrological) systems.* The obtained models depend only on two geometric parameters of the basin – area and elongation ratio – and can therefore be applied to ungauged watersheds. The models are parsimonious, involving two categories of nonlinear terms: (1) interaction terms that couple rainfall input with the current hydrological state of the watershed, and (2) autonomous terms that represent the internal dynamics of runoff.

These models are based either on single basins, or on multiple basins considered simultaneously, in association (specifically model G_I is based on four basins and model G_{II} on seventeen). Both demonstrate wide applicability across basins covering an area of ~223,000 km² in the Peruvian Pacific slope and coast, provided that basin areas and elongation ratios remain within moderate ranges (mainly small to medium, moderately elongated basins). This indicates that, beyond differences in rainfall regimes, watershed responses in this region follow a remarkably consistent dynamical pattern, *based on catchment morphology.*

These results highlight the potential of this modelling framework as a practical tool for hydrological prediction in data-scarce regions. Future work should test the transferability of these models to other geographic contexts, particularly to arid basins subject to infrequent but extreme rainfall events, in order to assess their robustness under broader hydrological conditions.

Conflict of interest statement

We have no conflict of interest to disclose.

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Data availability statement

The data that support the findings of this study can be made available on reasonable request.

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Appendix A: Simulations

In order to analyse how watershed responses vary with area and elongation ratio under different rainfall conditions, we performed simulations with the generalized model G_{II} using multiple forcings and different parameterizations of p_e and p_a . The structure of this model (Eq. 4) and its predictive performances will be considered in Section 5. Before that, our objective is to investigate the model response under different basin parameterizations and forcing conditions (Section A.1) and to characterize its stability (Section A.2).

A.1 Real forcings

We selected basin B22 as reference for this analysis because it is intermediate in terms of area ($p_a = 4,334 \text{ km}^2$), shape ($p_e = 0.51 \text{ km/km}$), precipitation (Table S1), and latitude (Figure 1). Six hydrological years were considered: three with average precipitation (2013-2014, 2014-2015, and 2018-2019), the first and third showing monomodal rainfall distributions and the second a bimodal one; two with low annual precipitation (2015-2016 and 2017-2018); and one with very high precipitation (2016-2017).

We first investigated the parameter range of validity. The model for basin B22 was found stable for areas between 600 and 7000 km^2 and elongation ratios between 0.4 and 0.65 km/km . Simulations were, then, performed to examine the influence of p_e and p_a on runoff under different rainfall conditions, first fixing p_a to 4,334 km^2 and varying p_e across its extremes, and then fixing p_e at 0.51 km/km while varying p_a .

A.2 Stability condition

We define the stability condition as the precipitation level required to maintain the discharge at a given level. This condition can be directly deduced from equation (4) by imposing $\dot{R} \equiv 0$, which corresponds to equilibrium in the runoff dynamics. In this case, the relation reduces to the precipitation P needed to sustain equilibrium:

$$P = f(R, p_e, p_a), \quad (5)$$

and, conversely, to the precipitation needed:

$$R = f(P, p_e, p_a). \quad (6)$$

The stability condition was evaluated for basin areas ranging from 0 to 18,000 km^2 and elongation ratios between 0 and 1. Although the calculations involve R and P , the final results are presented in terms of runoff ρ (R^2) and precipitation ϖ (P^2).

First, only one parameter was varied while keeping the other unchanged ($p_a = 4,334 \text{ km}^2$ or $p_e = 0.51 \text{ km/km}$). Second, simulations were also performed varying both parameters under different precipitation levels.

833 TABLES:

834 Table 1. Basic geometrical traits of the forty-nine basins considered in the analysis.

Number	Abrv.	Name	Area p_a (km ²)	Length (km)	Elongation ratio p_e (m ² /m ²)
B01	PIU	Piura	11,019.4	236	0.50
B02	CSJ	Cascajal	3,993.3	156	0.46
B03	OLM	Olmos	1,082.3	125	0.30
B04	MOT	Motupe	3,694.3	114	0.60
B05	CHL	Chancay_Lambayeque	4,061	180	0.40
B06	ZAN	Zaña	1,762.8	108	0.44
B07	CHM	Chaman	1,356.4	75	0.55
B08	JEQ	Jequetepeque	3,968.8	149	0.48
B09	CHI	Chicama	4,529.5	153	0.50
B10	MCH	Moche	2,131.9	93	0.56
B11	VIR	Viru	1,926.3	85	0.58
B12	HMN	Huamansana	1,439.5	65	0.66
B13	SAN	Santa	11,661.5	295	0.41
B14	LCR	Lacramarca	842.5	63	0.52
B15	NEP	Nepena	1,889.2	81	0.61
B16	CAS	Casma	2,990.7	95	0.65
B17	CLB	Culebras	670.9	53	0.55
B18	HMY	Huarmey	2,245	98	0.55
B19	FOR	Fortaleza	2,353	95	0.58
B20	PAT	Pativilca	4,601.7	160	0.48
B21	SUP	Supe	1,021.2	88	0.41
B22	HUA	Huaura	4,333.9	145	0.51
B23	CHH	Chancay_Huaral	3,062.6	100	0.62
B24	CHIL	Chillon	2,222.3	123	0.43
B25	RIM	Rimac	3,503.9	123	0.54
B26	LUR	Lurin	1,642.5	98	0.47
B27	CHLC	Chilca	783.4	45	0.70
B28	MAL	Mala	2,332.1	118	0.46
B29	OMA	Omas	1,117	65	0.58
B30	CAN	Canete	6,049.4	179	0.49
B31	TOP	Topara	620.3	58	0.48
B32	SJN	San Juan	3,353.2	135	0.48

B33	PIS	Pisco	4,231.2	172	0.43
B34	ICA	Ica	7,340.8	210	0.46
B35	GRN	Grande	11,049.9	138	0.86
B36	ACA	Acari	4,316	175	0.42
B37	YAU	Yauca	4,322.6	119	0.62
B38	CHA	Chala	1,232.4	65	0.61
B39	CHP	Chaparra	1,282.3	82	0.49
B40	ATI	Atico	1,400.3	71	0.59
B41	CRV	Caraveli	1,956.7	131	0.38
B42	OCO	Ocona	15,998.1	230	0.62
B43	CAM	Camana	17,152.8	355	0.42
B44	QUI	Quilca	13,548.6	220	0.60
B45	TMB	Tambo	13,073.4	295	0.44
B46	MOQ	Moquegua	3,419.1	130	0.51
B47	LOC	Locumba	5,862.2	175	0.49
B48	SAM	Sama	4,641.6	138	0.56
B49	CPL	Caplina	919.7	105	0.33

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837 *Table 2. Periods and basins used for calibration and validation.*

838

<i>Model structure</i>	<i>Data period</i>	<i>Calibration period</i>	<i>Validation period</i>	<i>Validation</i>	
				<i>Calibrated</i>	<i>Uncalibrated</i>
<i>Individual</i>	1970-2023	1970-2003	2003-2023	<i>All the basins:</i> 1-49	0 -
G_I	”	”	”	4 basins: 1-4	45 basins: 5-49
G_{II}	”	”	”	17 basins: 1,4,7,10,13,16,19, 22,25,28,31,34, 37,40,43,46,49	32 basins: 2,3,5,6,8,9,11,12, 14,15,17,18,20,21, 23,24,26,27,29,30, 32,33,35,36,38,39, 41,42,44,45,47,48

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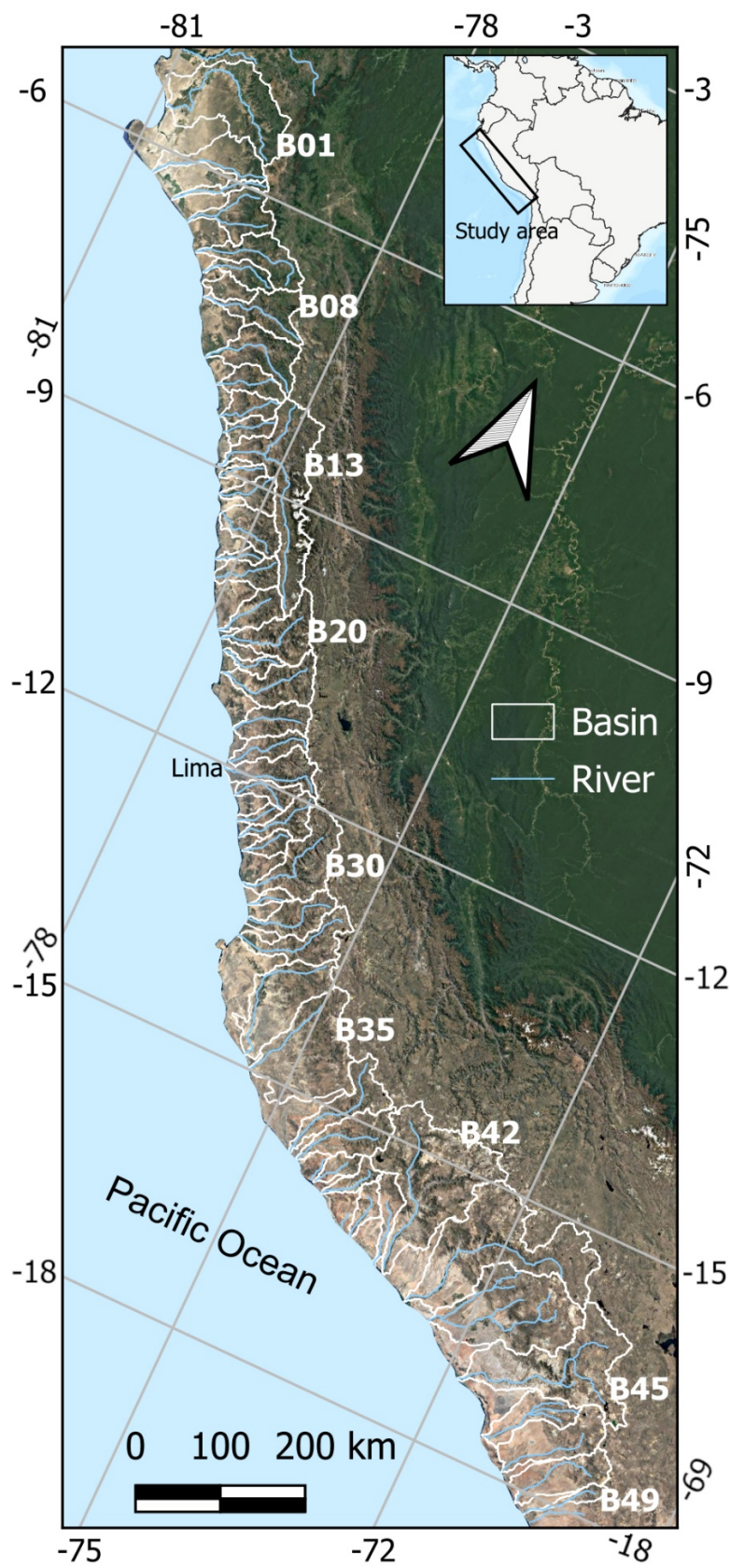
Table 3. Structures of the individual (a to s) and generalized (G_I and G_{II}) models. Ten basins from Piura (B01) to Moquegua (B46) share the same structure a, which is a parsimonious formulation with only three terms. Structure d, also found in ten basins spanning a wide latitudinal range, is similar to a but it replaces the negative R^2 term with a negative RP term. Structure o, shared by Ocona (B42) and Camana (B43), the two largest southern basins, includes a P^2 term but otherwise resembles structure a. Structure i occurs in six basins from Casma (B16) to Caraveli (B41) and it is a simplified version of a, lacking the term R^2P . Structure j (three basins) and k (two basins) are very similar, differing by only one term. All of these structures (a, d, o, i, j, k) are mutually compatible: terms may be present or absent, but shared terms always have the same sign. Structures f, g, h, l, n, and r occur in single model each but remain compatible with this group. In contrast, structures e (three models), m (two models) and p (one model) are incompatible with the previous group, as they contain positive RP and R^2P terms. Structures q and s, corresponding to two southernmost basins (B47 and B49), are unique, but compatible with each other. Similarly, B03 and B04 exhibit structures b and c, respectively; here c is compatible with both b and the e-m-p group, whereas b is not.

Individual models									
Structure	Monomials								Basins
	P^2	P^3	R	RP	RP^2	R^2	R^2P	R^3	
a					+	−	−		1,2,8,9,23,28,30,35,45,46
b		+	−	+	−	−	+	−	3
c				+		−	+	−	4
d				−	+		−		5,6,12,14,20,24,26,31,33,37
e					+	−	+	−	7,10,17
f				−	+	−		−	11
g					+	−		−	13
h				−	+	−	−		15
i					+	−			16,19,22,25,38,41
j		+		−	+			−	18,32,34
k		+		−	+		−		21,29
l		+	−						27
m				+		−	+		36,39
n		+		−	+				40
o	+				+	−			42,43
p				+	+	−			44
q					+	−	−	+	47
r		+		−	+				48
s				−	+		−	+	49
Generalized models									
Structure	Monomials								Basins
	P^2	P^3	R	RP	RP^2	R^2	R^2P	R^3	
G_I					+	−	+		1,4,7,8,9,10,11,12,14,16,17,18
					+	−	−		2,3,5,6,13,21,24,33,36,41,43,45,49
					−	−	+		27, 35
G_{II}					+	−	−	+	5,12,16,21,37,41:44
					+	−	−	−	4,6,13,23,24,26,33,36,38,45
					+	−	+	−	1,2,7:11, 14, 15, 17:20, 22, 25
					D/C	D/C	D/C	D/C	3,27,35,49

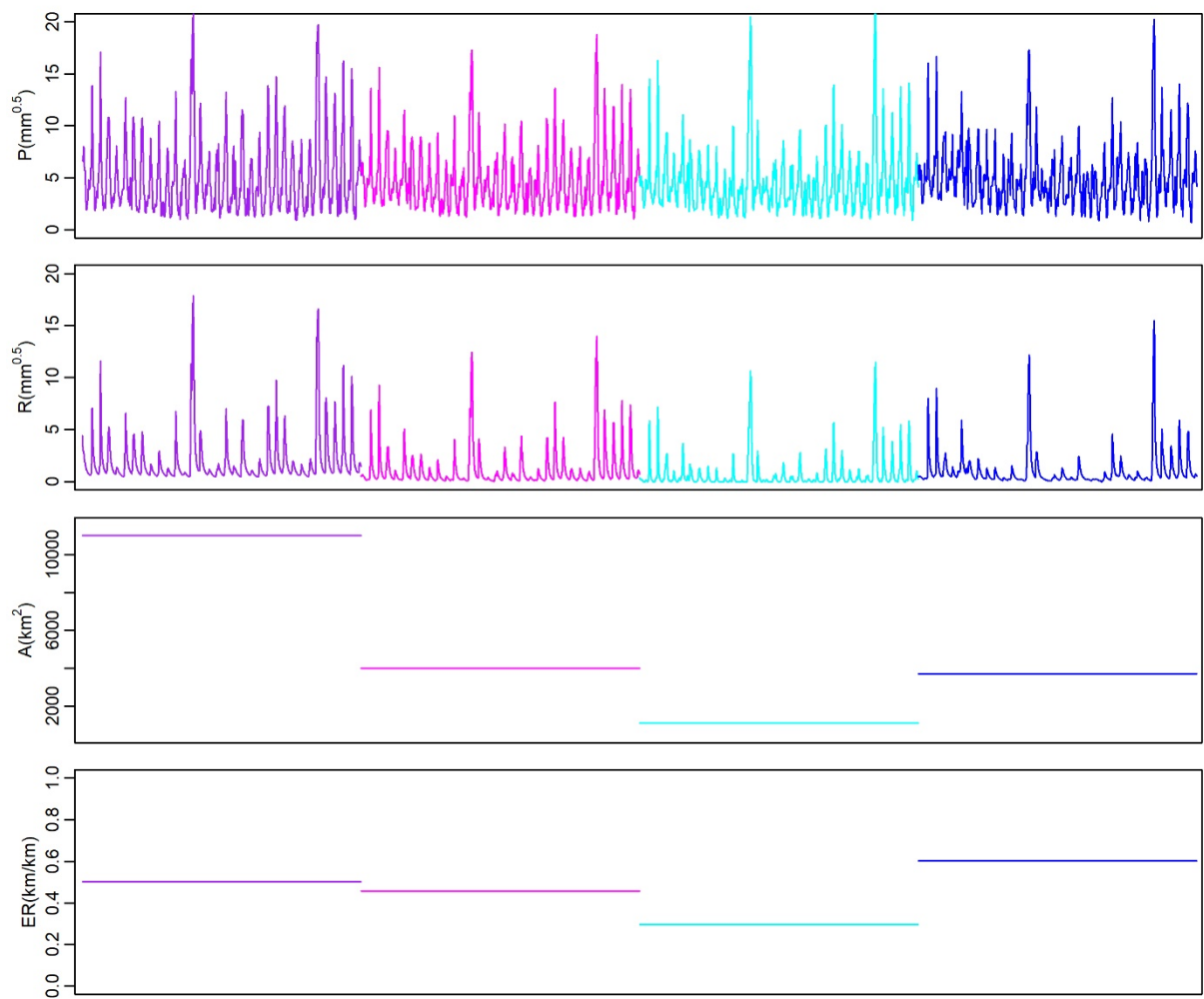
Table 4. Signs of the terms in Eq. (4) for each basin: terms in R^2P and p_e (2nd row); R^2P and p_a (3rd row); R^2P and $p_e p_a$ (4th row); R^3 and p_e (5th row); R^3 and p_a (6th row); R^3 and $p_e p_a$ (7th row).

G_{II}	B1, B4, B7, B9, B10, B11, B14, B15, B17, B18, B19, B22, B25, B29, B 30, B38, B39, B40, B44, B46, B47, B48	B8, B16, B20, B23, B26, B31, B32, B37, B42	B2, B5, B6, B12, B13, B21, B24, B28, B33, B34, B36, B41, B43, B45
$-R^2P(\beta_4$	+	—	—
$-\beta_5p_e$			
$+\beta_6p_e^2)$			
$-R^2P(-\beta_7p_a+\beta_9p_a^2)$	+	+	+
$-R^2P(\beta_8p_ep_a)$	—	—	—
$R^3(\beta_{10}-\beta_{11}p_e$	—	—	+
$+\beta_{12}p_e^2)$			
$R^3(-\beta_{13}p_a+\beta_{15}p_a^2)$	—	—	—
$R^3(\beta_{14}p_ep_a)$	+	+	+

862 FIGURES



863
864 *Figure 1. Region of study with the forty-nine coastal basins B01 to B49 in Peru.*



865

866 *Figure 2. Sketch of the dataset used for obtaining generalized models involving – at the same time – both*
867 *time series ($P(t)$ and $R(t)$, first two panels) and parameters (p_a and p_e , last two panels) from several*
868 *basins considered simultaneously. In this example, time series taken from basins B01 to B04 (in purple,*
869 *pink, cyan and blue colours) are concatenated in order to be processed in association (see [Mangiarotti et](#)*
870 *[al. 2016](#)). Note that during the modelling process, the discontinuities at the transition between different*
871 *basins will not be considered.*

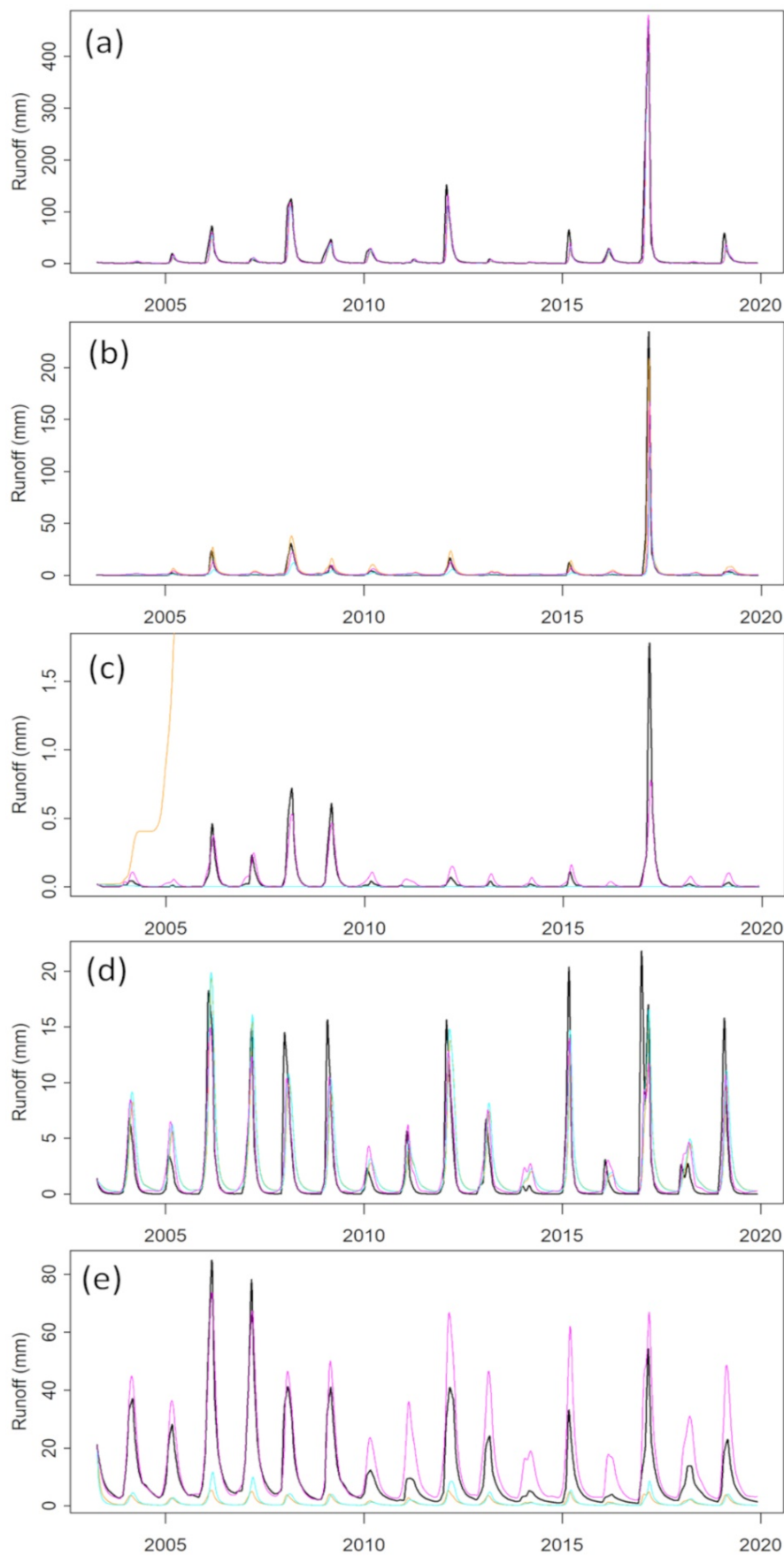
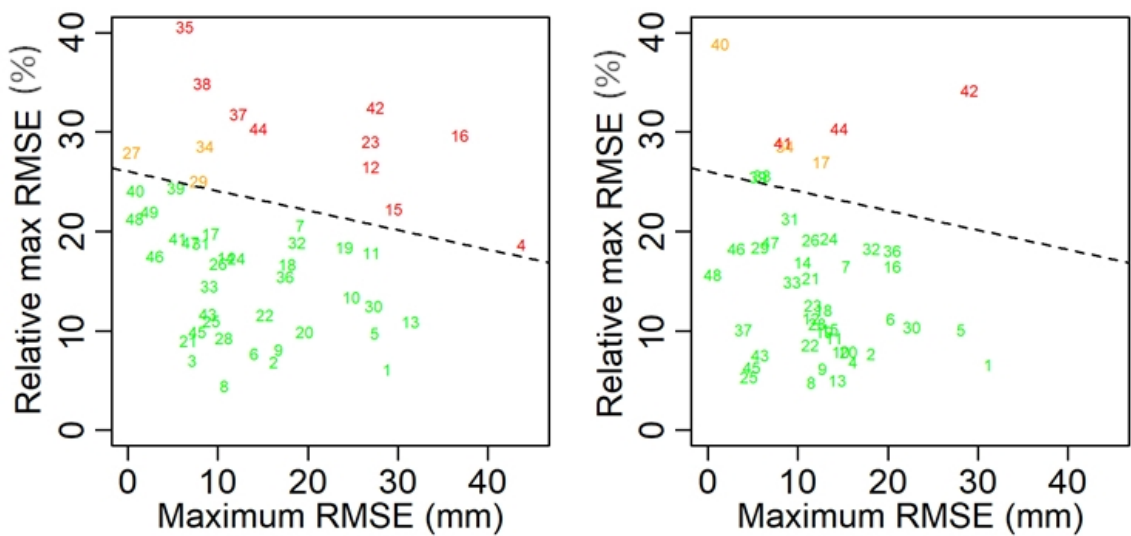
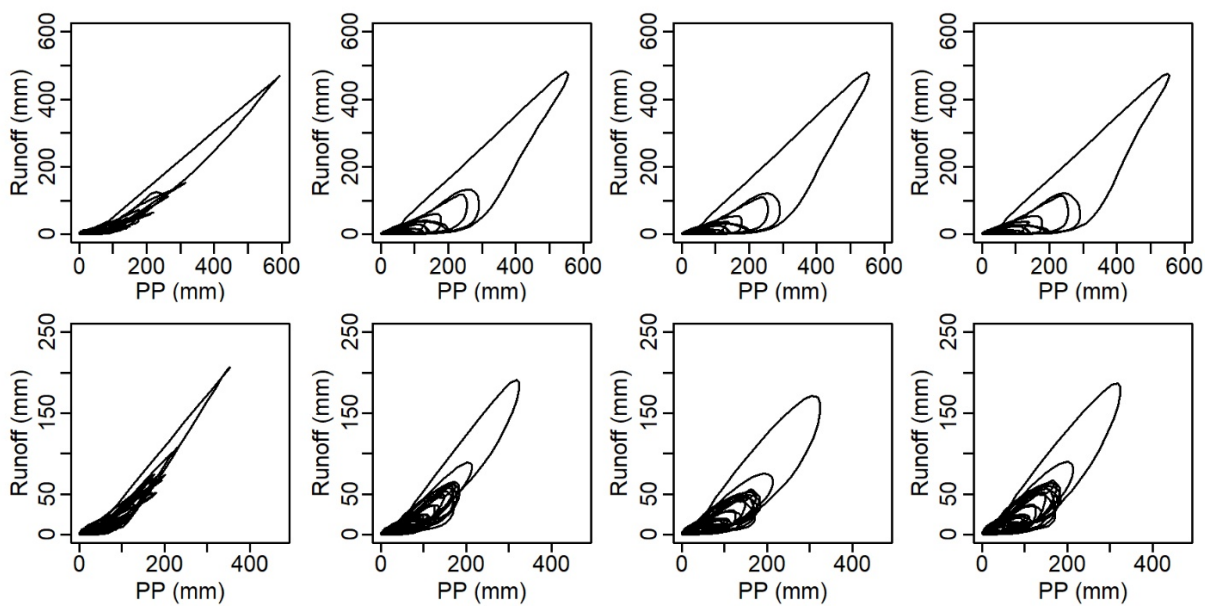


Figure 3. Original and predicted runoff $\rho(t)$ for basins B01 (a), B04 (b), B27 (c), B39 (d) and B42 (e). Original data (black), individual model (magenta), model G_I (cyan) and model G_{II} (orange).



875

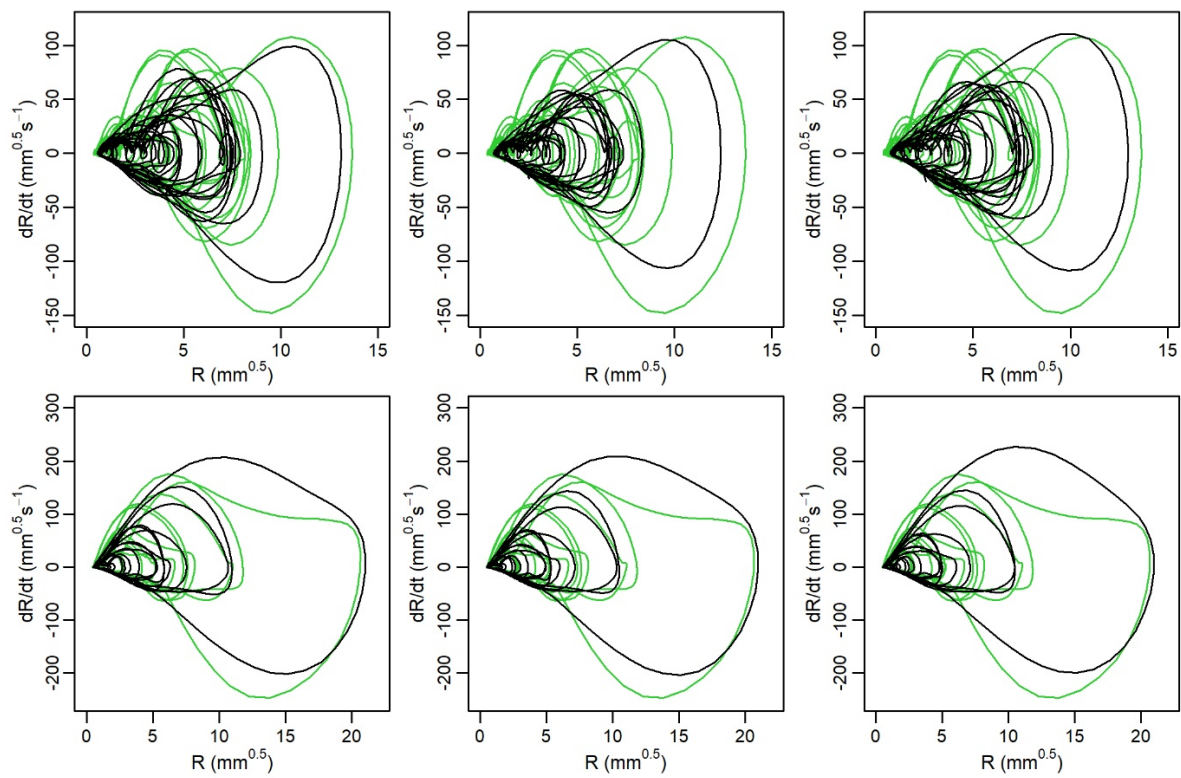
876 **Figure 4.** Relative maximum RMSE (in %) versus maximum RMSE (in mm) for models G_I (left panel) and
877 G_{II} (right panel). The numbers refer to the basins, and their performance is qualitatively classified (in green,
878 orange or red) comparing the predictions with the [data](#) in the scatter plots, time series and phase portraits.



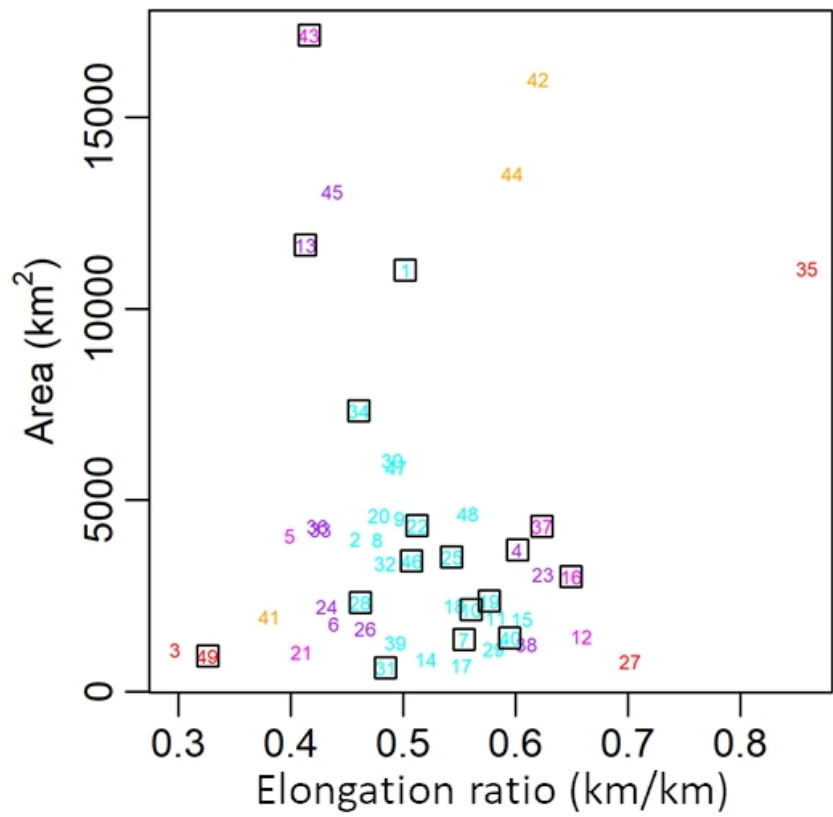
879

880 **Figure 5.** Original and models phase portrait (ρ , τ) for basins B01 (top panels) and B09 (bottom panels),
881 from left to right: [original data](#), individual model, G_I model and G_{II} model.

882

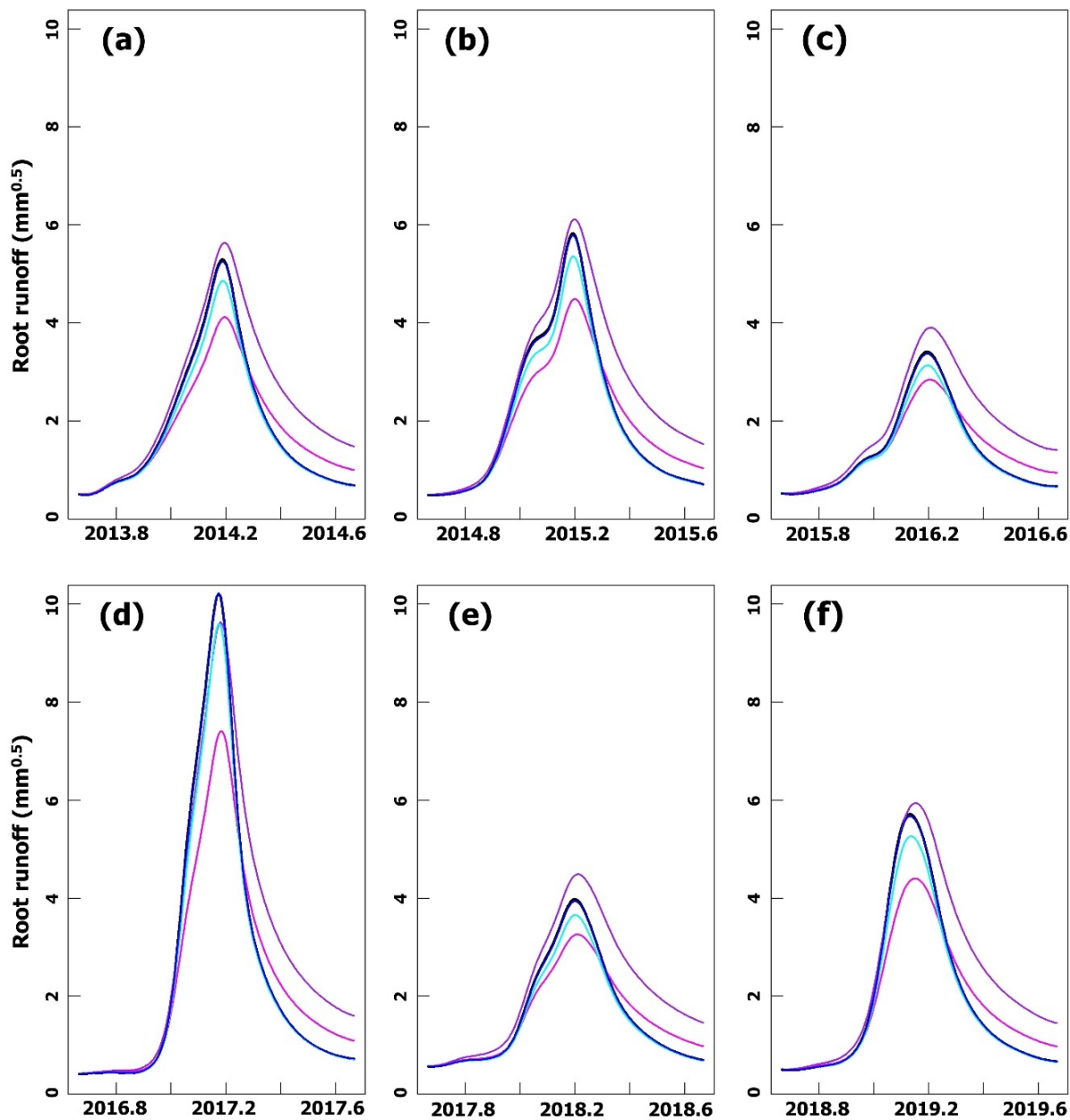


883 **Figure 6.** Models differential phase portrait (in black) of Runoff dynamics (R , dR/dt) for basins B01 (top
884 panels) and B09 (bottom panels), for individual model (left column), G_I model (centre column) and G_{II}
885 model (right column). Differential phase portraits reconstructed from the original time series are also
886 reported (in green) for comparison.



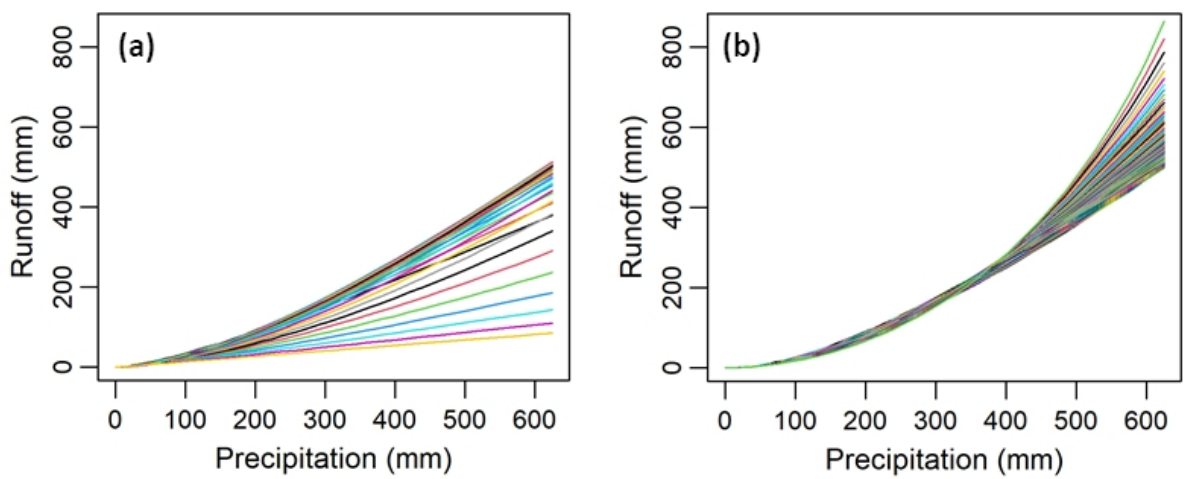
888

889 **Figure 7.** Ring distribution in the parameter space (p_e, p_a) of the basin parameterization type (in red,
890 orange, purple and cyan) considering the contributions to the terms in RP^2 , R^2P , R^2 and R^3 . The basins
891 rounded by a square have been used for the identification of the model structure $\mathbf{G}_{II}$, the other ones
892 have been used for validation.



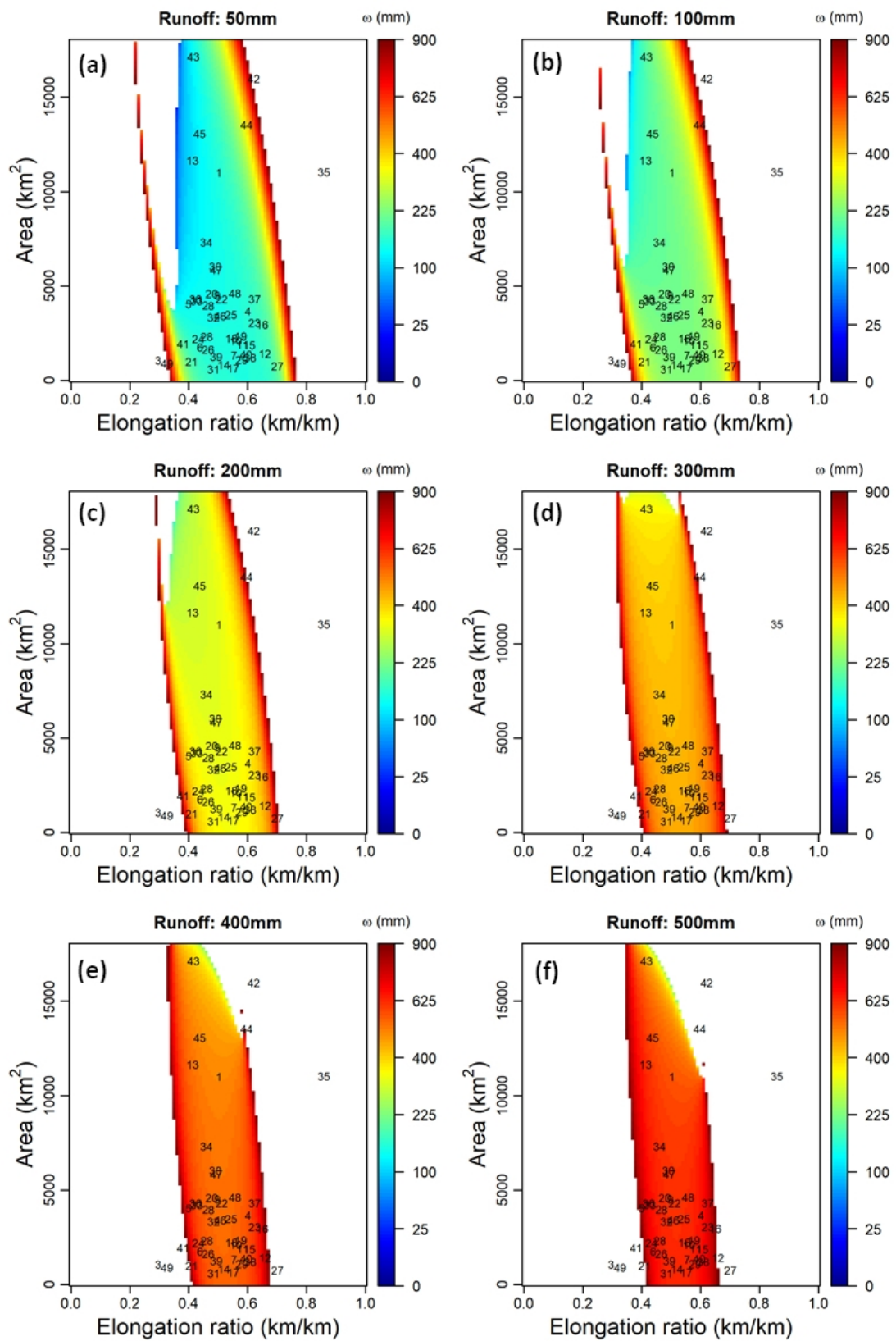
893

894 **Figure 8.** Simulations of the root runoff $R(t)$ with model $\mathbf{G}_{II}$ under different values of p_a and p_e for six
895 hydrological years from 2013-2014 (a) to 2018-2019 (f) successively with average / average / low / high
896 / low / average precipitation. Four simulations are presented in each case, first fixing the intermediate
897 area value to $p_a = 4,334$ km² and considering elongation values $p_e = 0.4$ km/km (in purple) for less
898 rounded to $p_e = 0.65$ km/km (in magenta) for more rounded; Then by fixing an intermediate elongation
899 value to $p_e = 0.51$ km/km and considering minimum $p_a = 600$ km² (in cyan) and maximum $p_a = 7,000$
900 km² (in blue) area values.



901

902 **Figure 9.** Dynamical stability condition for fixed average area ($p_a = 4,334 \text{ km}^2$) with elongation ratios p_e
903 varying from 0 to 1 km/km (a); And for fixed average elongation ratio ($p_e = 0.51 \text{ km/km}$) with areas p_a
904 varying from 0 to 18,000 km^2 (b).



905

906 *Figure 10.* Basin stability condition in precipitation ϖ (from 0mm in blue to 900mm in red) for different
907 combinations of area and elongation ratio, to get a certain level of runoff ρ from 50mm (a) to 500mm (f).