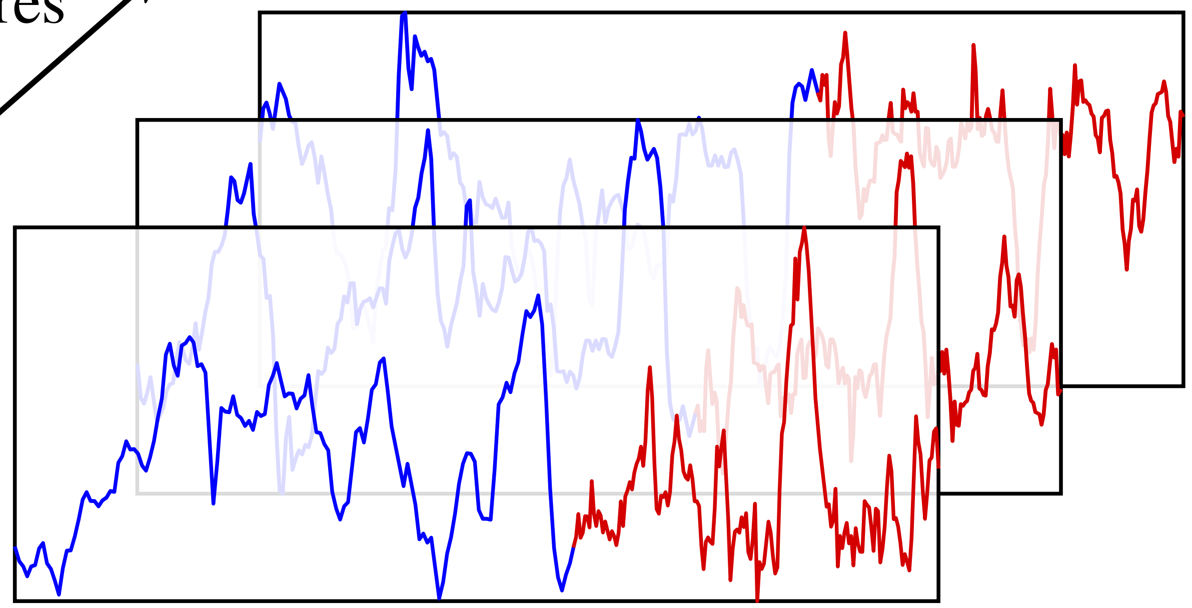
Overview for Physicists

Multiple realizations
or repeated measures
if ergodicity

series of observations ⇒ outcomes



Process

stochastic,
measurable

Causal states

series

$$S(x) = \{w : \mathrm{P}\left(future|past = w\right) \equiv \mathrm{P}\left(future|past = x\right)\}$$

Invariant by coordinate transforms

x and *w* ∈ S(x) have the same consequences

New observations cannot discriminate *x* vs *w*

State ≡ past information useful for predictions

"Equation of motion" on causal states → "dynamics of information"

New observation ⇒ new info. ⇒ Motion in causal state $S \in$ Hilbert space

Langevin type of dynamics on states $dS = \text{drift}\, dt + \text{diffusion}\, dW$

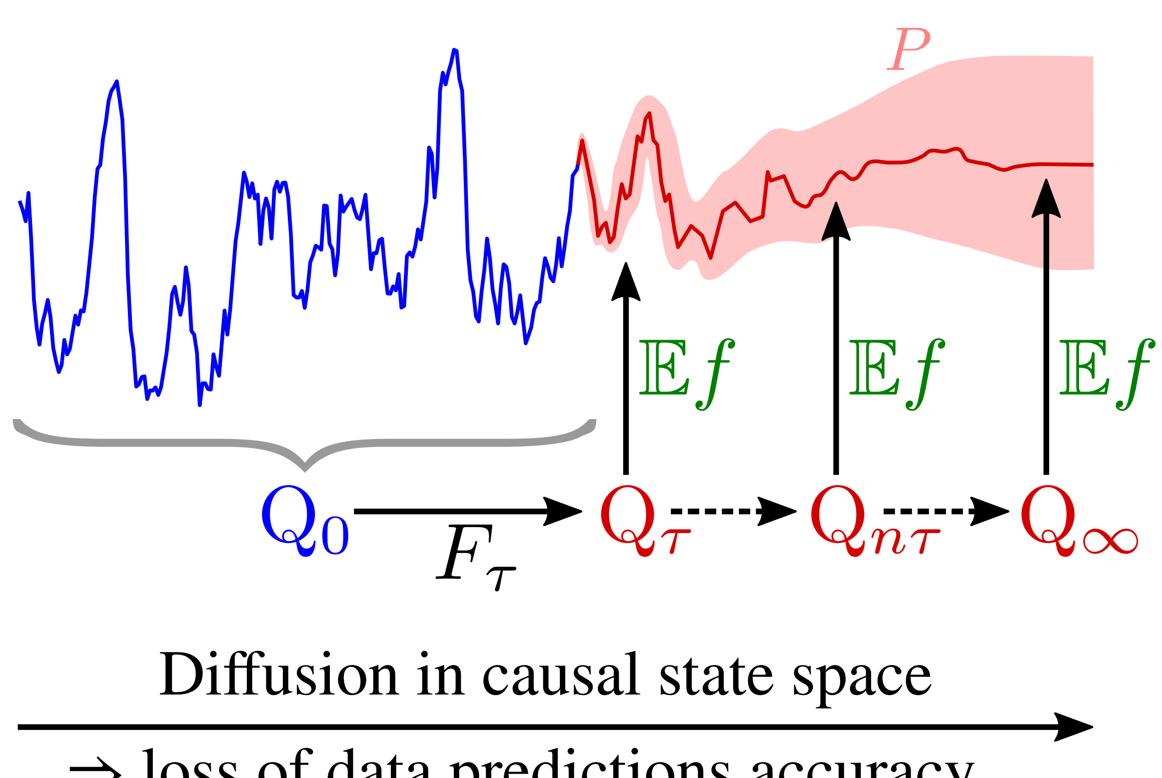
Fokker-Planck type of operator for distributions of states $\mathrm{Q}_{t+\tau} = F_\tau\left[\mathrm{Q}_t\right]$

Space of conditional distributions?
Continuity of trajectories?
Well-defined measure for Q?
⇒ Read the paper!
https://arxiv.org/abs/2011.14821

Predictive model

Estimated from data

- Initial distribution Q_0
  possibly $\mathrm{Q}_0 = \delta(\mathrm{S}_0)$ single state
- Evolution operator F_τ
- Expectation operator $\mathbb{E}f$
- Properties of the Process
  trajectories, attractor, densities,
  entropies, complexity measures…



$$\mathbb{E}f = \mathbb{E}_Q\left[\mathbb{E}_P\left[f(future)\right]\right]$$

f = function of the data

e.g. first value in the future series

Q_∞ = limit distribution of states
initial info. completely diffused