STRUCTURE AND FORCES IN STRESSED 3D PACKINGS

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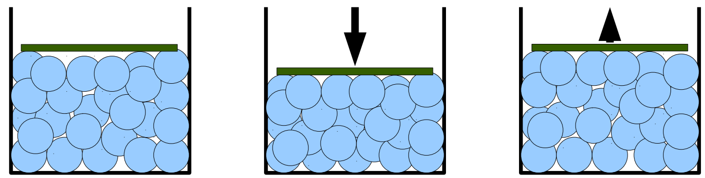
Rationale / Why

Mechanical stresses on granular assemblies are ubiquitous

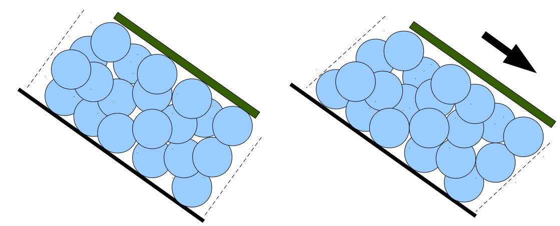
- Repeated compression

  (e.g. trucks passing on a road)



- Shearing

  (e.g. gravitational pull on a mountain slope)



- Industrial processes, civil engineering, environment...

Understanding their macroscopic response is necessary

- Requires “seeing” what is the state at the level of grains

Most studies are 2D. Most real cases are 3D.

This work = structure + forces in 3D

Accessing the micro-structure

X-rays / micro-ct
- Fine resolution
- Most materials
- Costly

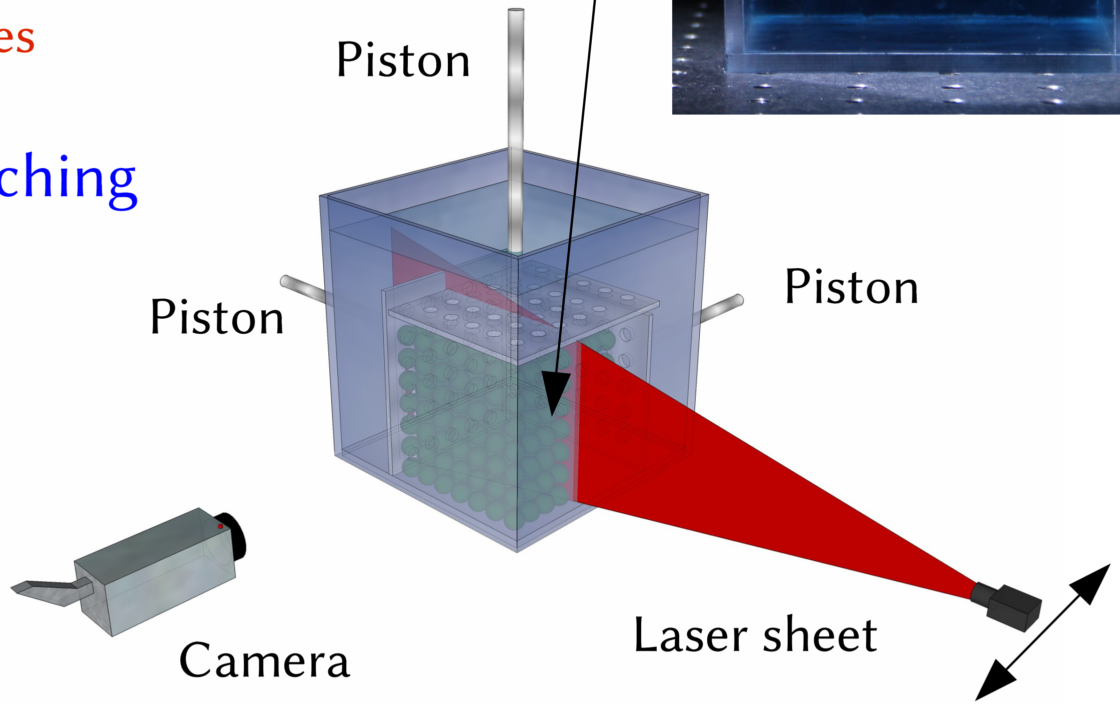
Confocal: emulsions
- Microscopic
- Costly
- Difficult to control applied stresses

This work: refractive index matching
- Macroscopic grains
- Easy to control, tri-axial shearing
- Cheap
- Submersed

Next slides on:
1. Structure
2. Forces in 3D

Mukhopadhyay *et al.*
Phys. Rev. E 84, 011302, 2011
Dijksman *et al.*
Rev. Sci. Intstrum. 2012

Hydrogel grains
index-matched
+ fluorescent dye



Typical image

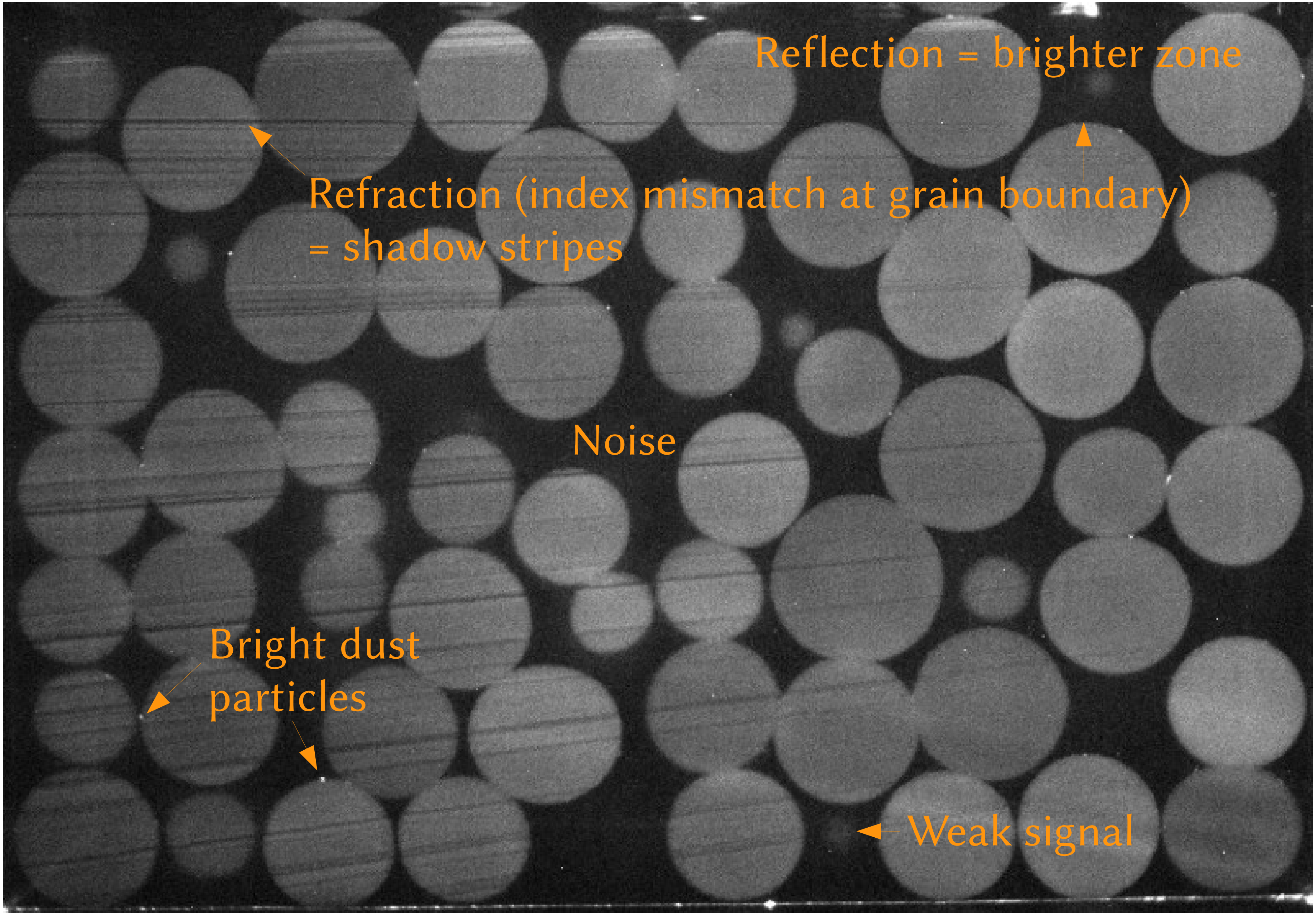
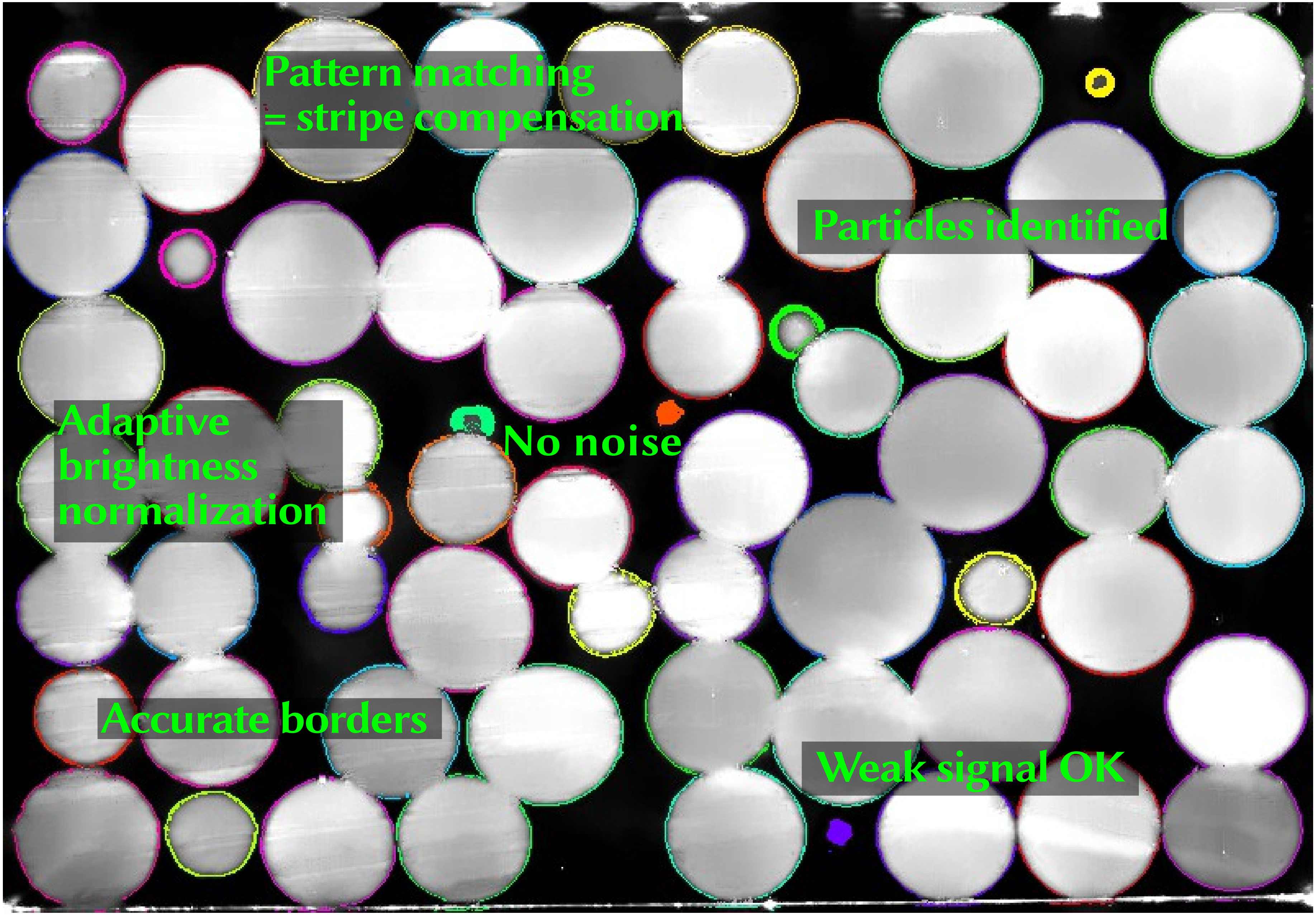


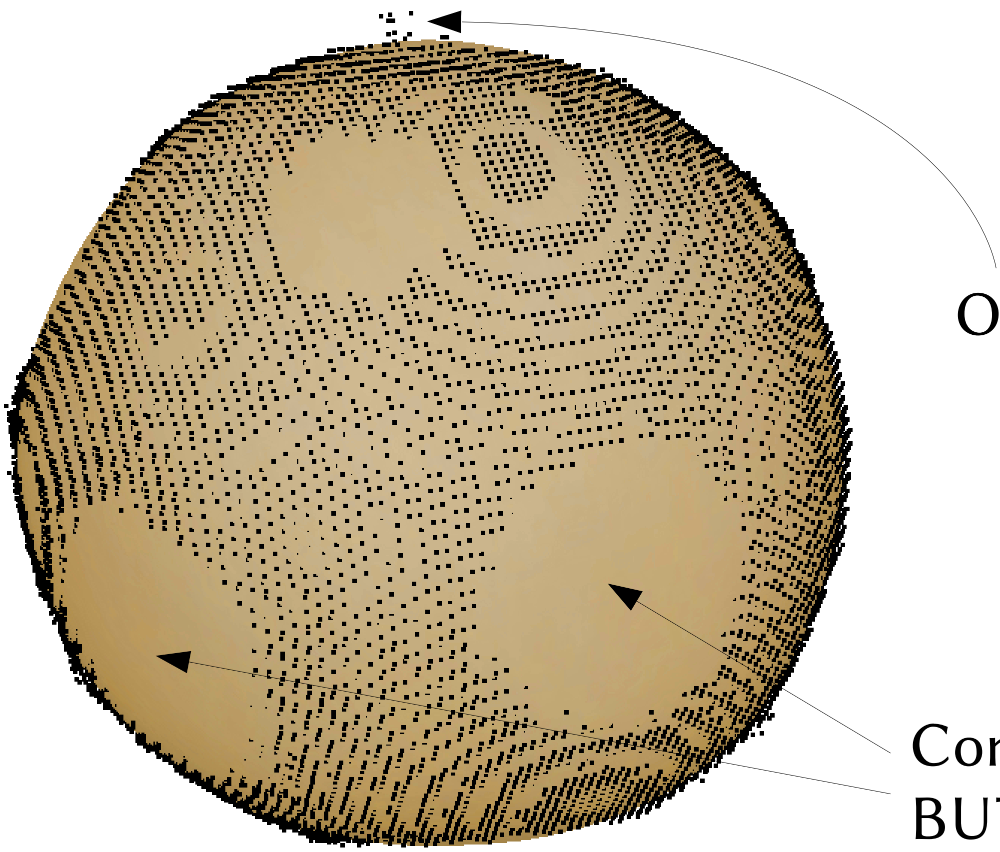
Image processing



Goal here = reliable structure information & reliable shapes ⟹ forces

From 2D images to 3D grains

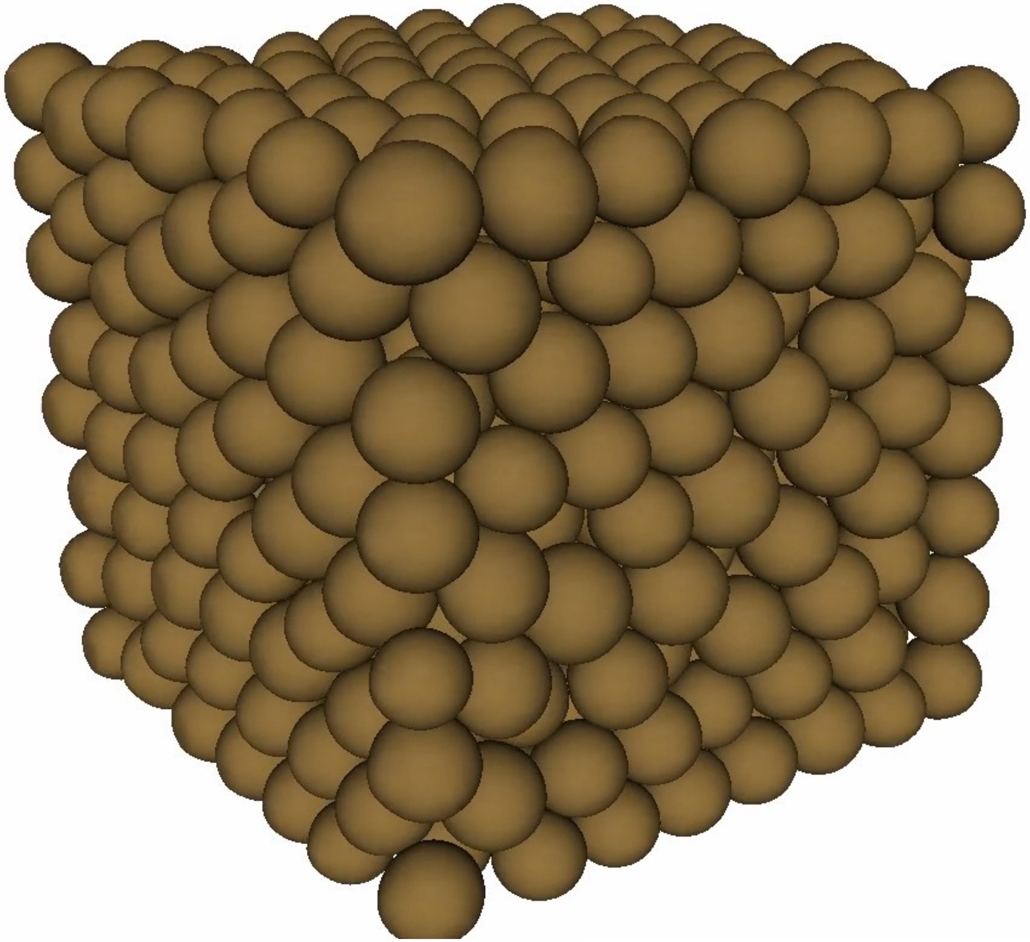
Step 1: Stack the images into 3D voxel
Step 2: Detect border voxels
Step 3: Fit an analytic surface to these borders
Done here using a spline basis of functions on the unit sphere
Step 4: Use these surfaces to get accurate forces

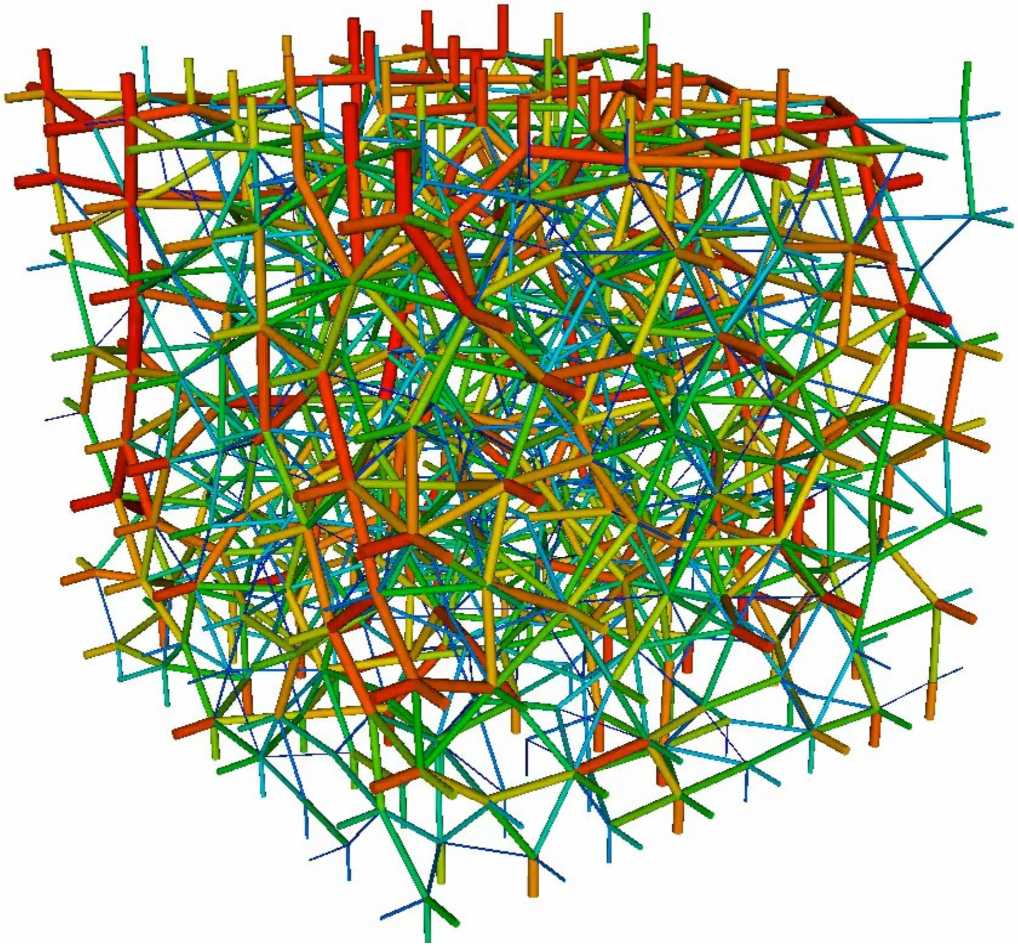


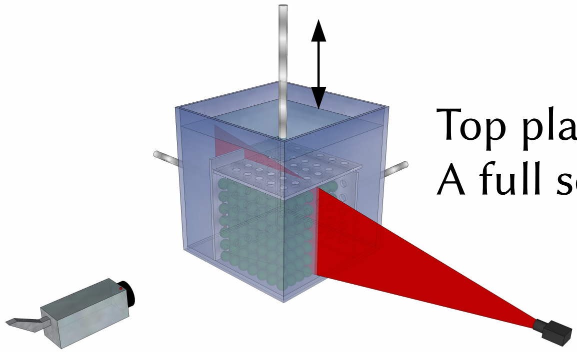
Outliers completely eliminated

Contacts = no border between grains
BUT surface area is well defined

10 uni-axial compression cycles





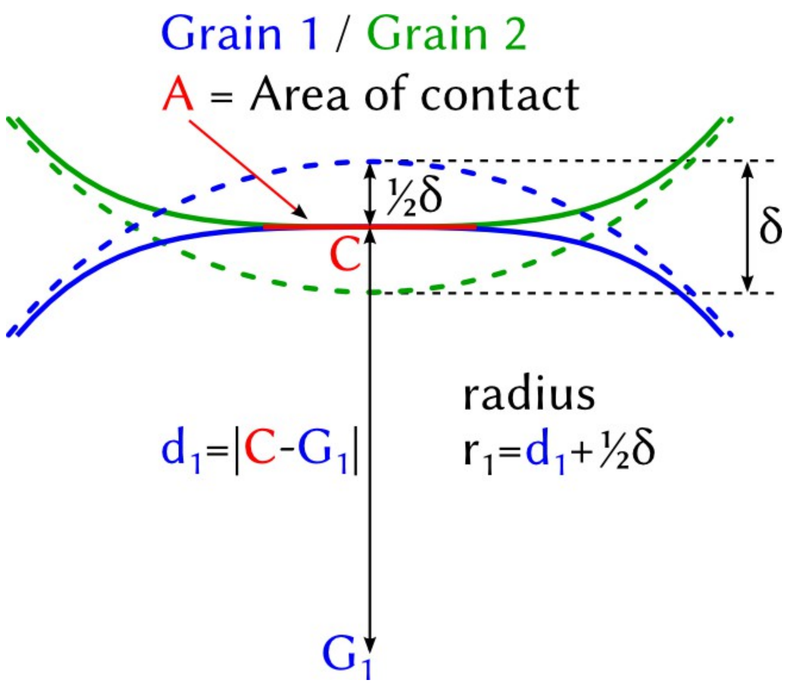


Top plate moves by 1mm increments
A full scan is taken between increments

Forces = struts joining the grain centers
Blue = weakest, Red = strongest

Inferring forces in full 3D

Analytic shape descriptions ⟹ contact properties



Measured here: A, C, G_1, G_2, d_1, d_2.
Unknown: δ

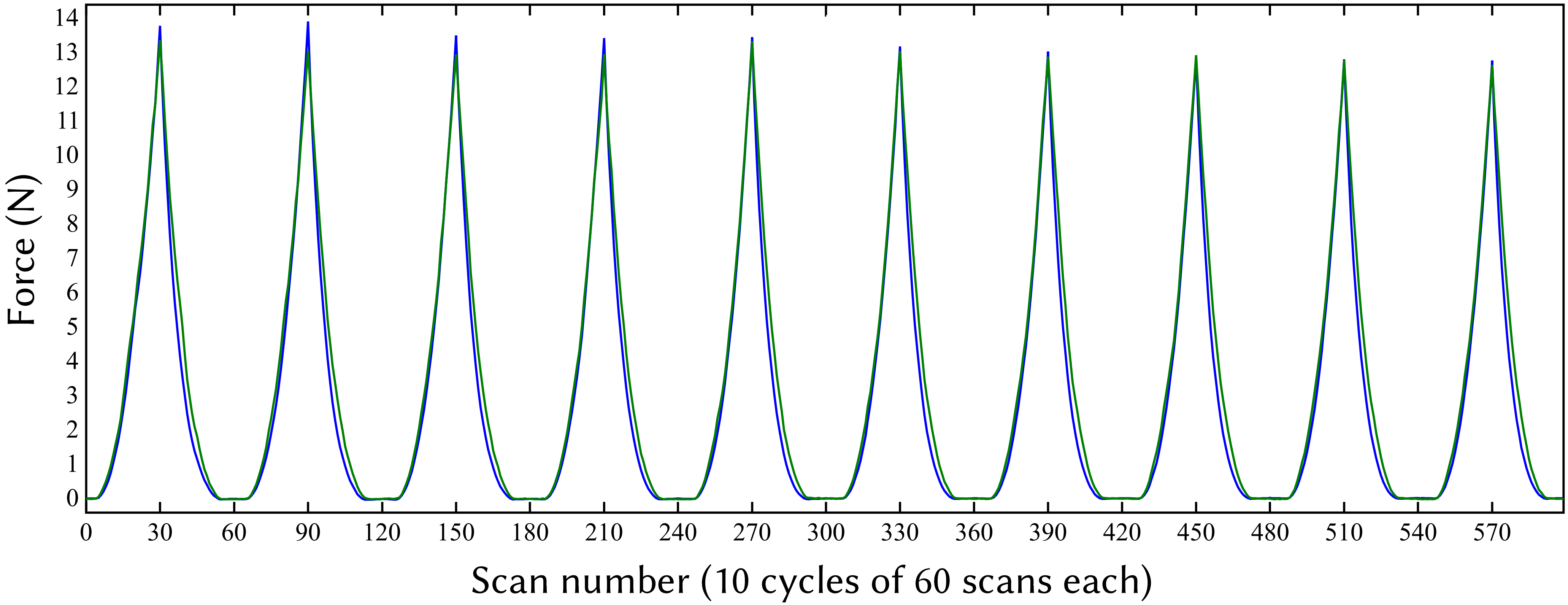
Contact properties ⟹ forces

$1/r = 1/r_1 + 1/r_2$	radius of curvature at contact
$F = E\, r^{1/2}\, \delta^{3/2}$	E = effective Young modulus
$F = E\, \delta\, a$	$a = \sqrt{A/\pi}$ radius of the contact

Hence $r\,\delta = A/\pi$ ⟹ δ ⟹ F (with given E)

⟹ Vector forces in full 3D, with orientation, position, norm
+ grain center, structure tensor, etc. all available.

Validation on compression cycles



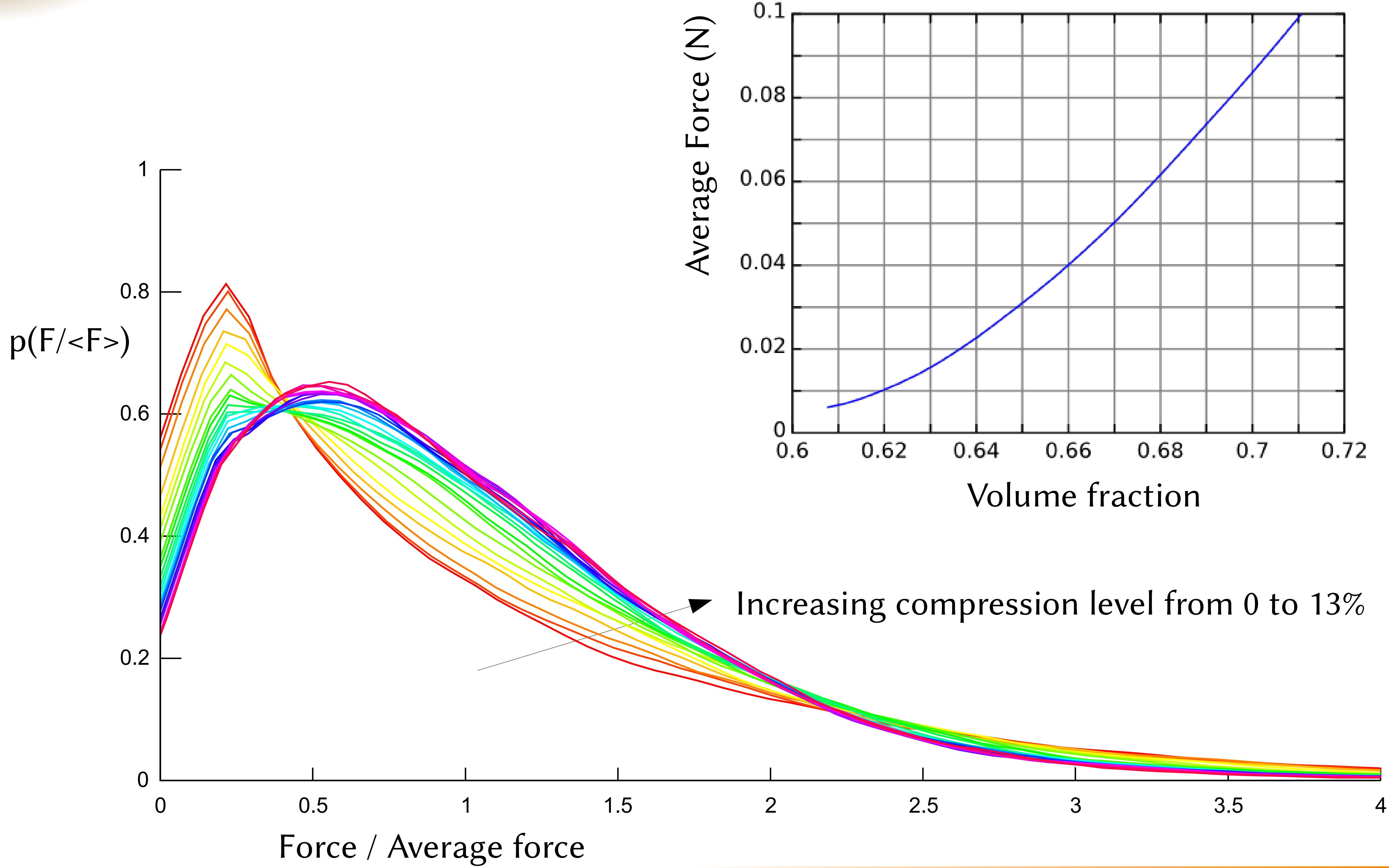
Blue = force measured on the top plate sensor

Green = force inferred from the images + measure of E=22.4 kPa

Grain deformation up to ≈13%, scan processed independently: no global fitting

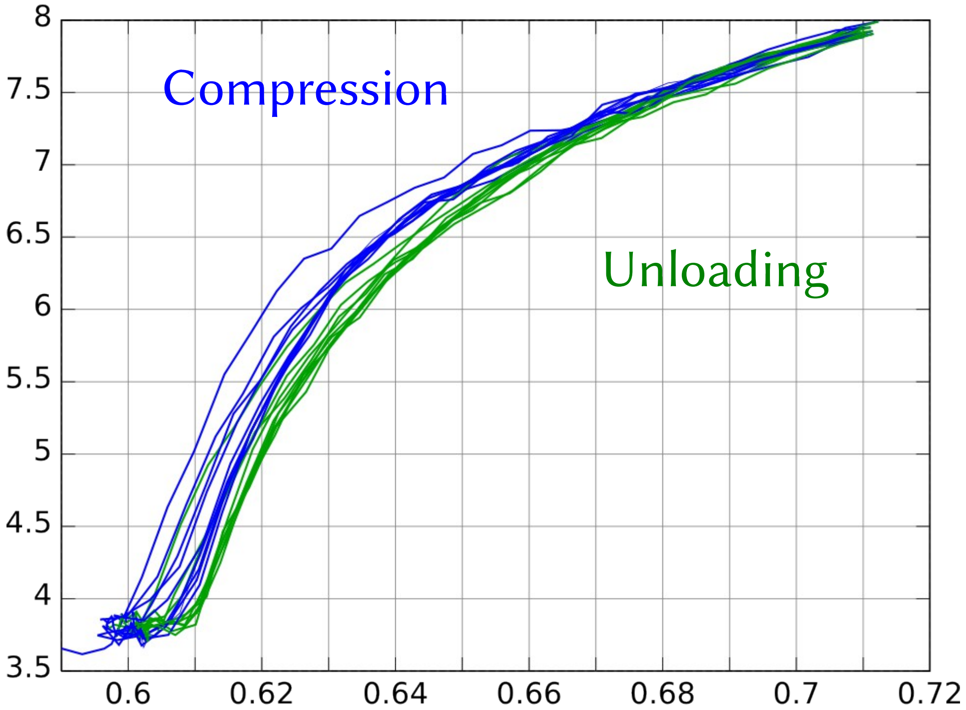
≈ $980 \cdot 10^3$ contacts over 600 scans. Resolution ≈10^{-2} N.

Force distribution



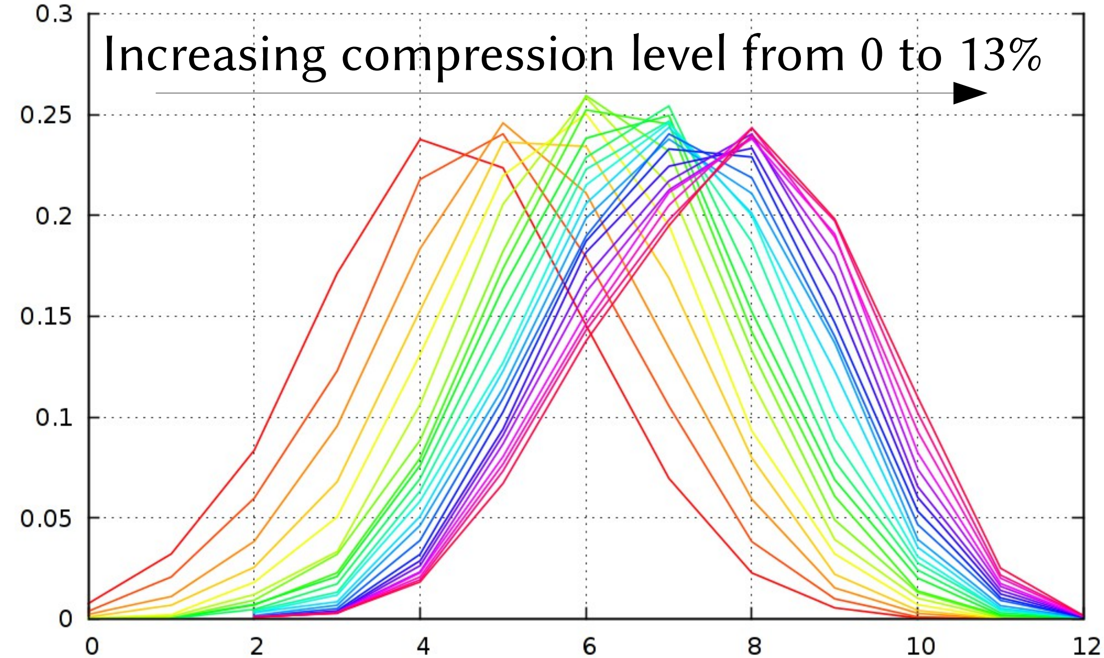
Number of Contacts

Number of contacts per grain (Z)



Volume fraction φ

p(Z)



Number of contacts per grain (Z)

Low friction + low effective gravity => small Z at small φ
Wide transition / no clear point for jamming

Conclusion

Accurate measurements of 3D forces now possible

- Extension to other contexts like micro-ct?

Access to the full micro structure while stressing the granular assembly

- 3D force chains/graph evolution while shearing/compressing?

Future works

– Topological analysis of the forces

– Structure of the force network: chains, stable configurations, etc.